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FURTHER MATHS TRIAL EXAM 2 SOLUTIONS 2004

Section A – Core - solutions

Question 1

a. Use the statistical function on your calculator to find these 1-Variable stats.
Mean = 17.7, standard deviation = 3.2
(1 mark) **(1 mark)**

b. **i.** From the box plot, we see that the shape of the data is symmetrical.
(1 mark)

ii. The median is 7.5.
(1 mark)

iii. 6 is the first quartile.
9 is the third quartile.
The percentage of data that lies between the first and the third quartiles is 50%, hence the percentage of data that lies between 6 and 9 is 50%.
(1 mark)

c. To fit a least squares regression line it has been assumed that the relationship is linear.
(1 mark)

d. Again, using a calculator, the equation of the least squares regression line is
$\text{number of incoming emails} = 6 \cdot 24 + 1 \cdot 51 \times \text{number of outgoing emails}$
(2 marks)

e. If the number of outgoing emails was 15, then
number of incoming emails

$$= 6 \cdot 24 + 1 \cdot 51 \times 15$$
$$= 28 \cdot 89$$
$$= 29 \text{ (to the nearest whole number)}$$

(1 mark)

f. For student F the actual number of incoming emails was 12. The number predicted by the least squares regression line is
$6 \cdot 24 + 1 \cdot 51 \times 5$ (where 5 is the number of outgoing emails for student F)
$= 13 \cdot 8$ (to 1 decimal place)
So the least squares regression line overestimates the number of incoming emails by $13 \cdot 79 - 12 = 1 \cdot 8$ emails.
(1 mark)

g. **i.** Using a calculator $r = 0 \cdot 776$

(1 mark)

ii. 60% of the variation in the number of incoming emails can be explained by the variation in the number of outgoing emails since the coefficient of determination is 0.60 (to 2 decimal places)

(1 mark)

Question 2

a.

Type of email(s)	**Gender of student**	
	Male	Female
Outgoing email(s)	**35%**	**52%**
Incoming email(s)	**65%**	**48%**
Total	100%	100%

(1 mark)

b. By percentaging the table in this way, we are comparing males with females.

(1 mark)

c. From Table 3, which has been percentaged, we note that males don't send emails as much as females but tend to receive more emails than females.

(1 mark)
Total 15 marks

Module 1: Number Patterns and Applications

Question 1

a. Vertical height of sixth step above the ground floor
$= 120 + 150 + 150 + 150 + 150 + 150$
$= 870\text{mm}$

(1 mark)

b. $t_n = a + (n-1)d$ where a is the first term in the sequence. The distance above the floor of the first step is 120mm. So $a = 120$. **(1 mark)**
The difference in height between the subsequent steps is 150mm.
So, $d = 150$. **(1 mark)**
(Do a check, $t_n = 120 + (n-1) \times 150$
So $t_2 = 120 + (2-1) \times 150$
$= 120 + 1 \times 150$
$= 270$
which is the height of the second step above the ground floor.
If you have time, check t_3 and so on.)

c. Method 1
Using the formula from part b.

$$\begin{aligned} t_n &= 120 + (n-1) \times 150 \\ &= 120 + 150n - 150 \end{aligned}$$

So $t_n = 150n - 30$
When $t_n = 3270$

$$\begin{aligned} 3270 &= 150n - 30 \\ 3300 &= 150n \\ n &= 22 \end{aligned}$$

There are 22 steps.

(1 mark)

Method 2
The first step is 120mm high.
Now $3270 - 120 = 3150$
The remaining steps are all 150mm high. So $3150 \div 150 = 21$.
There are in total $21 + 1 = 22$ steps.

(1 mark)

d. We are summing the vertical distance above the ground floor of each of the first 10 steps of the staircase.
From the formulae sheet,

$$S_n = \frac{n}{2}(2a + (n-1)d)$$ **(1 mark)**

so,

$$\begin{aligned} S_{10} &= \frac{10}{2}(2 \times 120 + (10-1) \times 150) \\ &= 5(240 + 9 \times 150) \\ &= 5(240 + 1350) \\ &= 7950 \end{aligned}$$

The total length of timber required is 7950mm.

(1 mark)

Question 2

a. Method 1

Vertical rise of second step

$= 1\cdot 05 \times 100$

$= 105\text{mm}$

(1 mark)

Method 2

Vertical rise of second step

$= 100 + 5\% \text{ of } 100$

$= 100 + \frac{5}{100} \times 100$

$= 100 + 5$

$= 105\text{mm}$

1 mark)

b. The vertical rise r_n of the n^{th} step is a geometric sequence with $a = 100$ and $r = 1\cdot 05$.

The general term for a term of a geometric sequence is

$t_n = a(r)^{n-1}$

In our case,

$r_n = 100(1\cdot 05)^{n-1}$ as required.

(1 mark) for correct explanation

c. From part b., $r_n = 100(1\cdot 05)^{n-1}$

When $r_n = 190$,

We have $190 = 100(1\cdot 05)^{n-1}$

$1\cdot 9 = (1\cdot 05)^{n-1}$

We need to solve this for *n*.

Method 1

Trial and error using a calculator

$1\cdot 05^{n-1} = 1\cdot 9$

Let $n = 5$,	$1\cdot 05^4 = 1\cdot 21...$	too low
Let $n = 10$,	$1\cdot 05^9 = 1\cdot 55...$	too low
Let $n = 20$,	$1\cdot 05^{19} = 2\cdot 52...$	too high
Let $n = 15$,	$1\cdot 05^{14} = 1\cdot 97...$	too high
Let $n = 14$,	$1\cdot 05^{13} = 1\cdot 88...$	just under

(1 mark)

The number of steps that can be legally constructed is 14. Since the 14^{th} step will be just under the legal limit. The 15^{th} step would be just over the limit.

(1 mark) for answer

Method 2
Use logs.

$$1 \cdot 05^{n-1} = 1 \cdot 9$$

$$\log_{10}(1 \cdot 05)^{n-1} = \log_{10}(1 \cdot 9)$$ **(1 mark)**

$$(n-1)\log_{10}(1 \cdot 05) = \log_{10}(1 \cdot 9)$$

$$n - 1 = \frac{\log_{10}(1 \cdot 9)}{\log_{10}(1 \cdot 05)}$$

$$n = \frac{\log_{10}(1 \cdot 9)}{\log_{10}(1 \cdot 05)} + 1$$

$$= 14 \cdot 16 \text{ (to 2 decimal places)}$$

The number of steps that can be legally constructed is the next whole number below 14.16 which is 14.

(1 mark)

Question 3

a. $R_{n+1} = 0 \cdot 98 R_n$ where $R_1 = 115$

$$R_1 = 115$$

$$R_2 = 0 \cdot 98 \times R_1 = 0 \cdot 98 \times 115 = 112 \cdot 7$$

$$R_3 = 0 \cdot 98 \times R_2 = 0 \cdot 98 \times 112 \cdot 7 = 110 \cdot 446$$

So the vertical rise of the third step is 110.446 mm. **(1 mark)**

b. Each term in the sequence described by the difference equation $R_{n+1} = 0 \cdot 98 R_n$, where $R_1 = 115$ is 0.98 times the value of the previous term.
So, there is a common ratio of 0.98. This sequence is therefore a geometric sequence.

(1 mark) for answer
(1 mark) for explanation

c. Because the pattern in the step height is described by a geometric sequence, we use the sum of an infinite geometric sequence.

$$S_n = \frac{a}{1-r}$$

$$S_\infty = \frac{115}{1 - 0 \cdot 98} = 5750$$

The total vertical height of the staircase would be 5750mm.

(1 mark) recognition of need for sum of geometric series formula
(1 mark) correct answer
Total 15 marks

Module 2: Geometry and Trigonometry

Question 1

a. In $\Delta ABD, (AB)^2 = (AD)^2 - (BD)^2$

$= 640^2 - 512^2$

$= 147\,456$

$AB = 384\text{m}$

(1 mark)

b. Area of $\Delta ABD = \frac{1}{2} \times AB \times BD$

$= \frac{1}{2} \times 384 \times 512$

$= 98\,304$ square metres

(1 mark)

c. In ΔABD, $\sin(\angle BAD) = \frac{512}{640}$

$\angle BAD = 53^\circ 8'$

as required.

(1 mark)

d. Since $\angle BAD = 53^\circ 8'$, then

$\angle ADB = 89^\circ 60' - 53^\circ 8'$

$= 36^\circ 52'$

Now, $359^\circ 60' - 36^\circ 52' = 323^\circ 8'$

$= 323.13^\circ$

So the bearing of A from D is 323.13°.

(1 mark)

e. In ΔBCD, $\angle BDC = 45^\circ$

Method 1

So ΔBCD is an isosceles triangle since $\angle BCD = 45^\circ$.

Hence $BC = BD = 512\text{m}$ **(1 mark)**

Method 2

In ΔBCD, $\tan(\angle BDC) = \frac{BC}{BD}$

$\tan 45^\circ = \frac{BC}{512}$

$1 = \frac{BC}{512}$

$BC = 512\text{m}$

(1 mark)

Question 2

a. Since BE is parallel to CD, ΔABE is similar to ΔACD. ($\angle CAD$ is common, $\angle ABE = \angle ACD$ (alternate angles) and $\angle AEB = \angle ADC$ (alternate angles))

So, $\dfrac{AE}{AD} = \dfrac{AB}{AC}$ **(1 mark)**

$\dfrac{AE}{640} = \dfrac{384}{384+512}$ (from Question 1 parts a. and e.)

$AE = 274 \cdot 29$

$= 274\text{m}$ (to the nearest metre)

(1 mark)

b. $\angle AEB = \angle ADC$ since ΔABE is similar to ΔACD.

So $\angle AEB = \angle CDB + \angle ADB$

$= 45^\circ + (90^\circ - 53^\circ 8')$

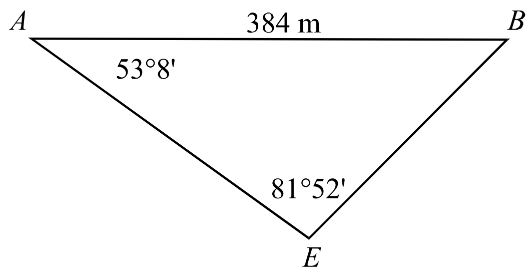
$= 45^\circ + 36^\circ 52'$

$= 81^\circ 52'$

(1 mark)

c. Hence means that you must use what you have just found.

In ΔABE



$\dfrac{BE}{\sin 53^\circ 8'} = \dfrac{384}{\sin 81^\circ 52'}$

$= 310\,\text{m}$ to the nearest metre

(1 mark) using the sine rule
(1 mark) correct answer

d. In ΔABE

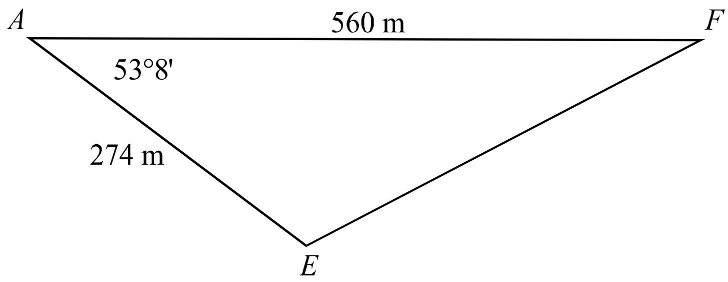
area $= \dfrac{1}{2} \times 384 \times 274 \sin 53^\circ 8'$ **(1 mark)**

$= 42\,088$ square metres (to the nearest square metre)

(1 mark)

Question 3

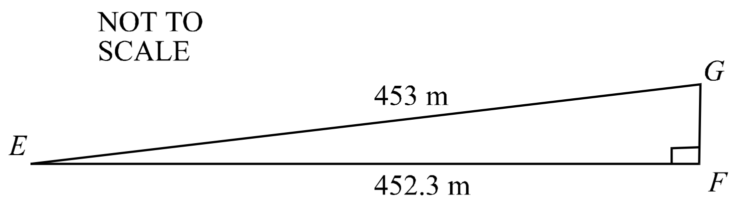
a. In ΔAFE,



$$(EF)^2 = 560^2 + 274^2 - 2 \times 560 \times 274 \cos 53^\circ 8'$$
$$EF = 452 \cdot 3\text{m} \text{ (correct to 1 decimal place)}$$

(1 mark) using the cosine rule
(1 mark) correct answer

b. In ΔEFG



$$(GF)^2 = 453^2 - 452 \cdot 3^2$$
$$GF = 25.2\text{ m} \text{ (correct to 1 decimal place)}$$

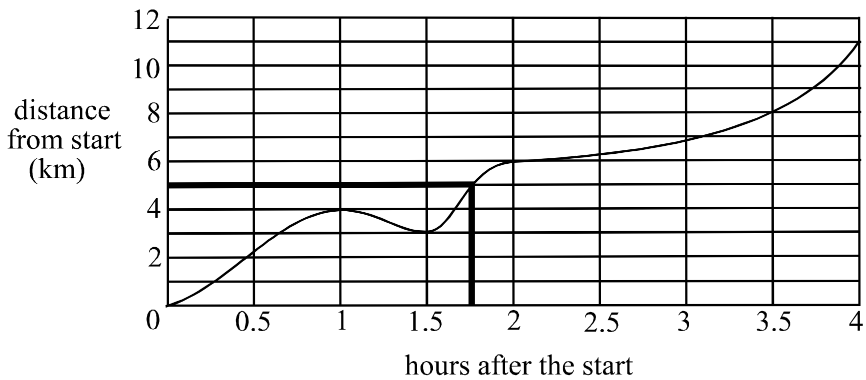
(1 mark)

The height of the old windmill is 25.2 metres (correct to 1 decimal place).

Module 3: Graphs and relations.

Question 1

a.



From the graph we see it took Mr Brown 1.75 hours to ride the first 5km of the course.

(1 mark)

b. Between $t = 2$ and $t = 3$, that is, during the third hour, Mr Brown covered the least distance. **(1 mark)**
This was because the gradient of the graph for that period was the least. **(1 mark)**

c. Between $t = 1$ and $t = 1 \cdot 5$ the distance between Mr Brown and the starting line of the walkathon decreases. Hence Mr Brown is riding back towards the starting line during this period.

(1 mark)

Question 2

a. The rests occur when the line is horizontal; that is the distance from the starting point of the walkathon is not changing.
The first rest is for ¼ hour and the second is for ¼ hour. In total there is ½ hour of rest.

(1 mark)

b. Kate is walking at her slowest when the gradient is least steep. Use a ruler to identify the least steep section of the graph. This occurs between $t = 1 \cdot 25$ and $2 \cdot 75$ when the gradient of the graph is $\frac{9-4}{2.75-1.25} = 3.\dot{3}$

(1 mark)

c. At $t = 3$, $d = 9$ and at $t = 3 \cdot 5$ and $d = 11$. **(1 mark)**
So the gradient of this section of the graph is

$$\frac{11-9}{3 \cdot 5 - 3} = \frac{2}{0 \cdot 5}$$
$$= 4$$

So Kate is walking at 4 km/hr between $t = 3$ and $t = 3 \cdot 5$.

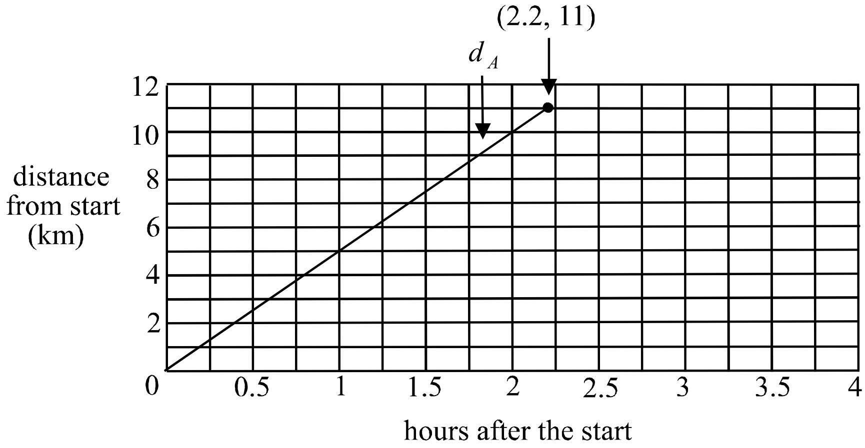
(1 mark)

d. From the graph, we see that Kate walks 11 km in the walkathon. So, she raises $11 \times \$12 = \$132$ in total for the walkathon.

(1 mark)

Question 3

a. **i.**



(1 mark)

ii. The course is 11km long (from Question 1 part d.), and for Alexa

$$d = 5t$$

So $11 = 5t$

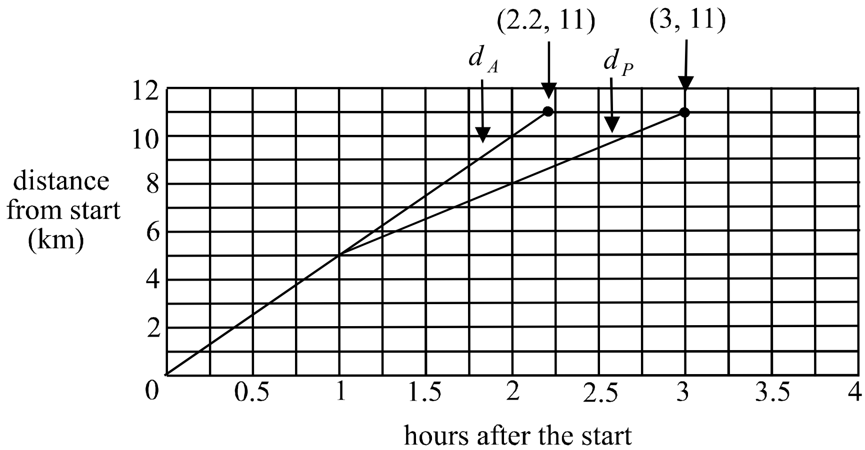
$$t = \frac{11}{5}$$

$$= 2 \cdot 2$$

So $m = 2 \cdot 2$.

(1 mark)

b. **i.**



(2 marks)

ii. For Peter, at the end of the walkathon, $d_p = 3t + 2$.

So $11 = 3t + 2$

$$9 = 3t$$

$$t = 3$$

So $n = 3$.

(1 mark)

c. From the graph in Question 1, Kate takes 3 ½ hours to finish the walkathon course. From Question 2 part a. ii., Alexa takes 2.2 hours. So Alexa beats Kate by 1.3 hours.

(1 mark)

Total 15 marks

Module 4: Business-related mathematics.

Question 1

a. 2.5% of $8500

$= \frac{2 \cdot 5}{100} \times 8500$

$= 212 \cdot 5$

You would pay $\$8500 - \$212 \cdot 5 = \$8287 \cdot 50$

(1 mark)

b. **i.** $\$99 \times 52 \times 2 = \$10\,296$

Interest paid $= \$10\,296 - \$8\,500$

$= \$1796$

(1 mark)

ii. $\left(\frac{1796}{8500} \times \frac{1}{2} \times \frac{100}{1}\right)\%$

$= 10 \cdot 564...\%$

$= 10.6\%$ (to 1 decimal places)

The annual flat rate of interest is 10.6%.

(1 mark) for omitting the half in the original line
(1 mark) for correct answer

Question 2

a. Value after 5 years $= \$20\,000 \times 0 \cdot 9^5$ (reducing balance method)

$= \$11809.80$

(1 mark) for correct type of depreciation
(1 mark) for correct answer

b. Value after 5 years $= \$20\,000 - 55\,000 \times \$0 \cdot 20$ (unit cost method)

$= \$9\,000$

(1 mark) for correct type of depreciation
(1 mark) for correct answer

Question 3

a. $A = 75\,000 \times 1 \cdot 005^{48}$

$= 95\,286 \cdot 69$ **(1 mark)**

Honest Harry would have earned $95 286.69 - $75 000 = $20 286.69

(1 mark)

b. We require

$100\,000 = 75\,000 \times 1 \cdot 005^n$

$1 \cdot \dot{3} = 1 \cdot 005^n$

(1 mark)

Method 1
Trial and error.

Let $n = 5$, $1 \cdot 005^5 = 1 \cdot 025...$ too low

Let $n = 10$, $1 \cdot 005^{10} = 1 \cdot 0511...$ too low

Let $n = 50$, $1 \cdot 005^{50} = 1 \cdot 283...$ too low

Let $n = 60$, $1 \cdot 005^{60} = 1 \cdot 348...$ too high

Let $n = 55$, $1 \cdot 005^{55} = 1 \cdot 315...$ too low

Let $n = 58$, $1 \cdot 005^{58} = 1 \cdot 335...$ too high

Let $n = 57$, $1 \cdot 005^{57} = 1 \cdot 3288...$ too low

It would take 58 months to reach (and pass a little) \$100 000. **(1 mark)**

Method 2
Use logarithms.

$$1 \cdot 005^n = 1.\dot{3}$$

$$\log_{10} 1 \cdot 005^n = \log_{10} 1 \cdot \dot{3}$$

$$n \log_{10} 1 \cdot 005 = \log_{10} 1 \cdot \dot{3}$$

$$n = \frac{\log_{10} 1 \cdot \dot{3}}{\log_{10} 1 \cdot 005}$$

$$= 57 \cdot 6$$

So it takes 58 months to reach (and pass a little) \$100 000. **(1 mark)**

Question 4

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

Given that the loan is paid off in 10 years and that $R = 1 \cdot 045$, we have

$$0 = 300\,000 \times 1 \cdot 045^{20} - \frac{Q(1 \cdot 045^{20} - 1)}{1 \cdot 045 - 1}$$

$$\frac{Q(1 \cdot 045^{20} - 1)}{0 \cdot 045} = 300\,000 \times 1 \cdot 045^{20}$$ **(1 mark)**

$$Q(1 \cdot 045^{20} - 1) = 0 \cdot 045 \times 300\,000 \times 1 \cdot 045^{20}$$

$$Q = \frac{0 \cdot 045 \times 300\,000 \times 1 \cdot 045^{20}}{1 \cdot 045^{20} - 1}$$

Note: not all the steps above are necessary; they are here to help understand how such a formula may be rearranged.

$$Q = 23\,062 \cdot 84$$ **(1 mark)**

So the amount owing after 5 years is given by

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

$$= 300\,000 \times 1 \cdot 045^{10} - \frac{23\,062 \cdot 84(1 \cdot 045^{10} - 1)}{1 \cdot 045 - 1}$$

$$= 182\,489 \cdot 82$$

There is \$182 489.82 owing after 5 years.

(1 mark)
Total 15 marks

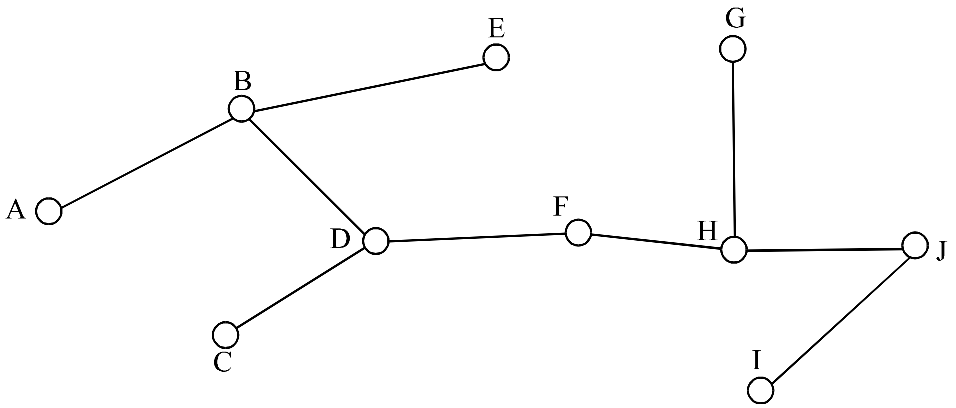
Module 5: Networks and business mathematics

Question 1

a. The shortest path from A to J is 225m long. ($ABEGJ$).

(1 mark)

b.



(2 marks)

Question 2

Using the Hungarian algorithm.
Step 1
Create a matrix from the table given.

$$\begin{bmatrix} 6 & 7 & 8 & 6 \\ 8 & 8 & 5 & 7 \\ 4 & 4 & 7 & 5 \\ 5 & 4 & 3 & 3 \end{bmatrix}$$

Step 2
Use row reduction to obtain the next matrix.

$$\begin{bmatrix} 0 & 1 & 2 & 0 \\ 3 & 3 & 0 & 2 \\ 0 & 0 & 3 & 1 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

(1 mark)

Step 3
The second row contains only 1 zero.
Box it and cross out any other zeros in that column.

$$\begin{bmatrix} 0 & 1 & 2 & 0 \\ 3 & 3 & \boxed{0} & 2 \\ 0 & 0 & 3 & 1 \\ 2 & 1 & \not{0} & 0 \end{bmatrix}$$

Step 4
The last row contains only 1 zero.
Box it and cross out any other zeros in that column.

$$\begin{bmatrix} 0 & 1 & 2 & \not{0} \\ 3 & 3 & \boxed{0} & 2 \\ 0 & 0 & 3 & 1 \\ 2 & 1 & \not{0} & \boxed{0} \end{bmatrix}$$

Step 5
The first row contains only 1 zero.
Box it and cross out any other zeros in that column.

$$\begin{bmatrix} \boxed{0} & 1 & 2 & \not{0} \\ 3 & 3 & \boxed{0} & 2 \\ \not{0} & 0 & 3 & 1 \\ 2 & 1 & \not{0} & \boxed{0} \end{bmatrix}$$

(1 mark)

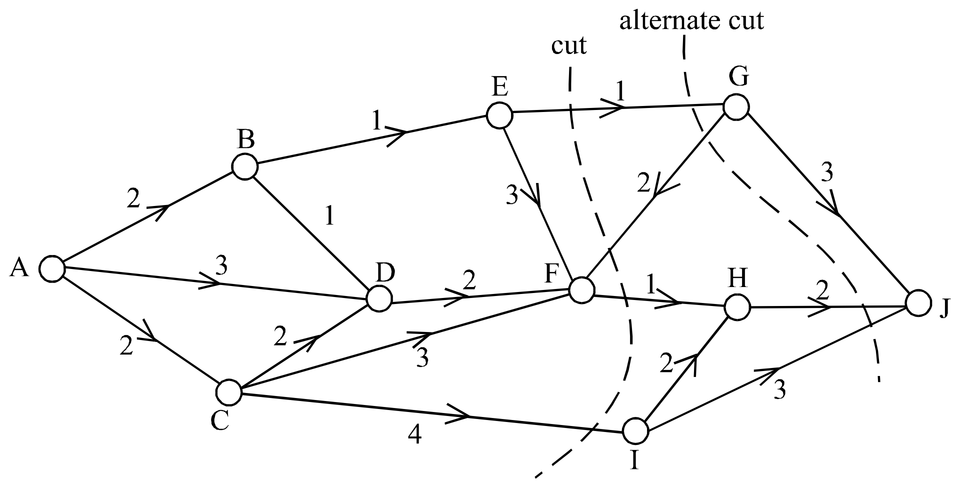
We have our optimal allocation.
Kate – construction costing
Monica – cash flow analysis
Gareth – financing
Ed – planning

(1 mark)

Question 3

a. The capacity of the cut shown is $1+1+2+3=7$.
Note that the arc *GF* is not counted since the water flows from *J* to *A* across the cut.
The value 2 is not counted (and certainly not subtracted!) **(1 mark)**

b. To find the maximum flow, we need to find the minimum cut. We do this by trial and error. There are two minimum cuts.
They are shown in the diagram below.

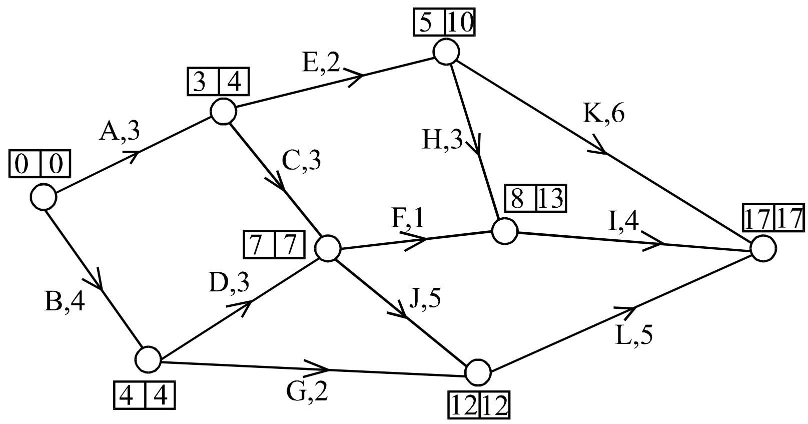


ks)

The first cut has a capacity of $1+1+4=6$. The alternate cut has a capacity of $1+2+3=6$.
Hence the maximum flow per hour is 6mL.

Question 4

a.



On the network above near each node are two numbers. The first is the earliest start time and the second is the latest start time.
The immediate predecessors of *I* are *F* and *H*. **(1 mark)**
The earliest start time for activity *L* is 12 weeks. **(1 mark)**
The latest start time for activity *G* is $12-2=10$ weeks. **(1 mark)**

b. The critical path is *B, D, J, L*. **(1 mark)**

c. Find the slack or float time for activities *G* and *F*.
Slack time for $G = 10 - 4 = 6$
Slack time for $H = 10 - 5 = 5$

(1 mark)

If both activities are indefinitely delayed then activity *H* has only 5 hours of slack time compared to 6 hours of slack time for activity *G*. So activity *H* will be first to delay the project. **(1 mark)**

Total 15 marks