# 2003 Further Mathematics
# Written Examination 2 (Analysis task)
# Suggested answers and solutions

## Core: Data analysis

### Question 1

**a** The male column must sum to 3458; so the number of men with 'normal' blood pressure is 2184. **[A1]**

**b** The two variables are *blood pressure* and *gender*. It is more likely that *blood pressure* depends on *gender.*

*Gender* is the independent variable. **[A1]**

**c** **i** $\frac{1672}{6504} \times 100 = 25.7\%$ **[A1]**

**ii** $\frac{930}{1672} \times 100 = 55.6\%$ **[A1]**

**d** The percentage of men with normal blood pressure is $\frac{2184}{3458} \times 100 = 63.2\%$

The percentage of men with high blood pressure is $\frac{930}{3458} \times 100 = 26.9\%$ **[M1]**

**e** Low blood pressure appears to be related to gender; 40.5%of the females had low blood pressure compared to only 9.9% of the males.

Evidence that high blood pressure is related to gender is not conclusive: 26.9% of the males compared to 24.4% of the females had high blood pressure; a difference of only 2.5%.

Blood pressure in the normal range seems to be related to gender; more men, 63.2%, had normal blood pressure compared to the 35.1% of females.

Either statement **[M1]**
Sensible figures quoted **[H1]**

### Question 2

**a** There is moderate, positive correlation between the variables. **[A1]**

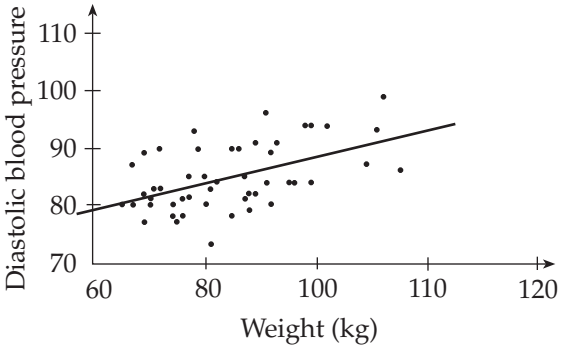
(This means that as weight increases, diastolic blood pressure also increases.)

**b** Diastolic blood pressure
$= 66.4 + 0.22 \times 80$
$= 84$ (mm Hg) **[A1]**

**c** Another point (100, 88.4)

Plot both (80, 84) and (100, 88.4) then draw the line through these two points.

Any two points plotted **[M1]**
Correct line **[A1]**




[Line also goes through (60, 79.6) and (110, 90.6)]

**d** Slope = 0.22 **[A1]**

For each kilogram increase in weight the diastolic blood pressure increases by 0.22 (units). **[A1]**

**e** Coefficient of determination = $r^2$

$r^2 = 0.5114^2 = 0.2615$ **[A1]**

"26.2% of the variation in diastolic blood pressure can be explained by the variation in weight." **[H1]**

**Total: 15 marks**

# Module 1: Number patterns and applications

## Question 1

**a** 1 : 19 : 7 ratio gives a total number of shares of 1+19+7= 27

The swim of 800 metres has 1 share therefore the run leg with 7 shares is equivalent to 5600 metres

( $7 \times 800$ m = 5600 m) **[A1]**

**b** The total distance of the race is
$27 \times 800$m = 21 600 m = 21.6 km **[A1]**

**c** cycle leg = 12 km

run leg = 6 km ( half the length of the cycle)

swim leg = 3 km ( the balance of 21 km)

gives a ratio

swim : cycle : run

3 km : 12 km : 6 km **[A1]**

simplifies to

1 : 4 : 2 **[A1]**

## Question 2

**a** Test for common ratio, $r = \frac{r_2}{r_1} = \frac{r_3}{r_2} = \frac{r_4}{r_3}$

Common ratio, $r\ \frac{r_2}{r_1} = \frac{6}{2} = 3$

and $\frac{r_3}{r_2} = \frac{18}{6} = 3$ **[M1]**

Hence it is a geometric sequence proven by the common ratio. **[A1]**

**b** If the common ratio = 3 **[A1]**

Then $r_4 = 3 \times r_3$

$= 3 \times 18 = 54$ **[A1]**

Checking $r_5 = 3 \times r_4$

$= 3 \times 54 = 162$

**c** $a = 2$ **[M1]**

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$S_8 = \frac{2\left(3^8 - 1\right)}{3 - 1}$$ **[M1]**

$S_8 = 6560$ **[A1]**

## Question 3

**a** Net loss of fluids for each kilometre

$b = (100 - 120) = -20$

initial body fluid content after first kilometre

$t_1 = 7\,800$ **[A2]**

so $t_{n+1} = t_n - 20$, where $t_1 = 7800$

**b** The general form of a difference equation with $t_{n+1} = t_n - 20$ ; $t_1 = 7\,800$ is

$t_n = t_1 + (n-1)b$

and for $a = 1$ (arithmetic)

$b = -20$; $t_1 = 7\,800$ **[M1]**

$t_n = 7800 - 20(n-1)$

$t_{25} = 7800 - 20(25-1)$

$t_{25} = 7800 - 4800 = 3000$ ml **[A1]**

or ALTERNATIVELY an iteration technique using the difference equation to generate 25 terms e.g. 7800, 7780, 7760, 7740, 7720 ……

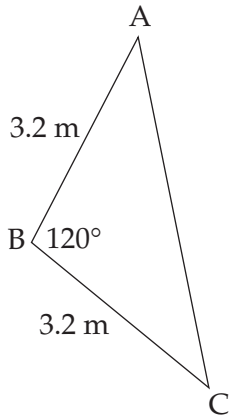
Therefore he should not continue the race in the hot conditions. **[A1]**

**Total: 15 marks**

# Module 2: Geometry and trigonometry

## Question 1

**a**




Using the cosine rule :

$\mathbf{AC}^2 = 3.2^2 + 3.2^2 - 2 \times 3.2^2 \cos 120°$

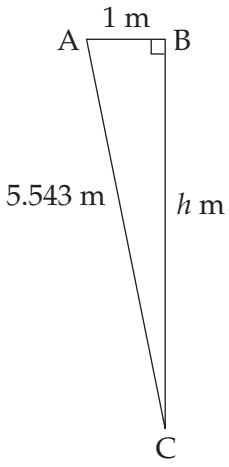
$= 30.75$ **[M1]**

**AC** = 5.543 m, correct to 3 decimal places.

**b** Area of $\Delta$ **ABC** $= \frac{1}{2} \times 3.2 \times 3.2 \times \sin 120°$ **[M1]**

$= 4.434\ \text{m}^2$ **[A1]**

**c**




$h^2 = 5.543^2 - 1^2$ **[M1]**

$h = 5.45$; to 2 decimal places **[A1]**

**d** Area of a trapezium $= \frac{1}{2} \times h \times (\overrightarrow{AF} + \overrightarrow{CD})$

$= \frac{1}{2} \times 5.452 \times (7 + 5)$

**[M1]**

$= 32.7122$ **[A1]**

**Total area of patio**

$= 2 \times$ area of $\Delta ABC$ + area of trapezium

$= 2 \times 4.434 + 32.712$

$= 41.6\ m^2$, to one decimal place. **[A1]**

### Question 2

**Ratio of lengths:**

Scale drawing : Actual = 1 : 50

**Ratio of areas:**

Scale drawing : Actual $= 1^2 : 50^2$ **[M1]**

$= 1 : 2500$

Hence the area ratio equation:-

$25.2 : x = 1 : 2500$ **[A1]**

where $x$ is the actual area of the sail.

$$\frac{x}{25.2} = \frac{2500}{1}$$

$x = 2500 \times 25.2$

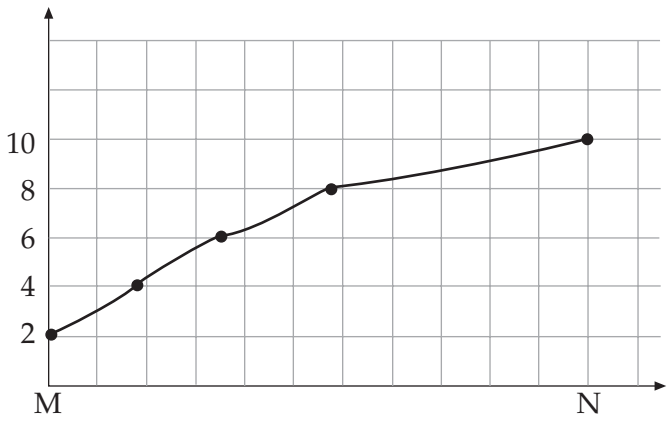
$= 63\ 000\ cm^2$

$63\ 000\ cm^2 = 6.3\ m^2$ **[A1]**

$(1m^2 = 100cm \times 100cm = 10\ 000cm^2)$

### Question 3

**a**




Scale **[A1]**

All correct **[A1]**

**b** Fencing$^2 = 42^2 + 8^2$ **[M1]**

Fencing = 42.755 metres

Need to allow 1 metre extra to account for the slope. **[A1]**

**Total: 15 marks**

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## Module 3: Graphs and relations

### Question 1

**a** From the graph

For COST

200 runners at \$44000

400 runners at \$50000

$$gradient = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{50000 - 44000}{400 - 200} = \frac{6000}{200}$$

= \$30 per runner pair **[A1]**

For REVENUE

200 runners at \$20000

400 runners at \$50000

$$gradient = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{50000 - 20000}{400 - 200} = \frac{30000}{200}$$

= \$150 per runner pair **[A1]**

| FUNCTION | Gradient | $y$-intercept |
|---|---|---|
| MANUFACTURING COST | \$ 30 per runner | \$38000 |
| RETAIL REVENUE | \$150 per runner | –\$10000 |

**[A1][A1]**

**b** Using simultaneous equations show that the break even point is 400 runners per week.

Break even point is when

COST = REVENUE

$30 \times$ number of runners + 38000

$= 150 \times$ number of runners – 10000 **[M1]**

$$30x + 38000 = 150x - 10000$$

$$38000 + 10000 = 150x - 30x$$

$$48000 = 120x$$

$$x = 400$$

400 runners need to be manufactured and sold each week **[A1]**

**c** For 450 runners

COST = $30 \times$ number of runners + 38000

$= 30 \times 450 + 38000$

$= 13500 + 38000$

$= 51500$

REVENUE = $150 \times$ number of runners – 10000

$= 150 \times 450 - 10000$

$= 67500 - 10000$

$= 57500$ **[M1]**

Profit = Revenue – Cost

$= 57500 - 51500$

$= \$6000$ **[A1]**

## Question 2

**a** Since 2-person unit cost \$50 per night, the total cost for 2-person units would be $50x$

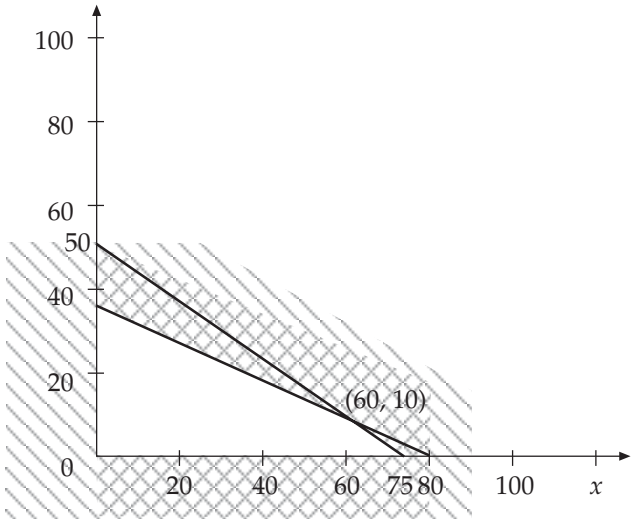
and for 3-person unit cabins the total cost is $100y$ with a total budget up to \$4000.

Therefore the budget constraint is

$50x + 100y \leq 4000$ **[A1]**

**b** $y \geq 0$ and $x \geq 0$ **[A1]**

**c**




for the $y \geq 0$ and $x \geq 0$ **[A1]**

for each of the two other constraints **[A2]**

**d** From the above graph, the three vertices are as follows

(0,40) NB: 0 units for 2 person and 40 units for 3 person (for 120 people and cost of \$4000)

(60,10) NB: 60 units for 2 person and 10 units for 3 person (for 150 people and cost of \$4000)

(75,0) NB: 75 units for 2 person and 0 units for 3 person (for 150 people and cost of \$3750)

for correct co-ordinates and **[A1]**

for correct costs. **[A1]**

**Total: 15 marks**

# Module 4: Business related mathematics

## Question 1

**a** $\$50\,000 + \$80\,000 = \$130\,000$ **[A1]**

**b** 5% of $\$80\,000 = \$4\,000$ **[A1]**

**c** $10 \times 4\,000 = \$40\,000$ interest
plus \$80 000 principal = \$120 000 **[A1]**

**d** Using the simple interest formula

$$I = \frac{Prt}{100} \Rightarrow 25000 = \frac{8000 \times r \times 5}{100}$$ **[M1]**

$r = 6.25\%$ **[A1]**

**e** $\$120\,000 - \$105\,000 = \$15\,000$ **[A1]**

## Question 2

**a** $n = 30 \times 12 = 360$ **[A1]**

$$R = 1 + \frac{\frac{6.5}{12}}{100} = 1.0054$$ **[A1]**

**b** Substituting correctly into the formula

$$0 = 130000 \times 1.0054^{360} - \frac{Q(1.0054^{360} - 1)}{1.0054 - 1}$$ **[M1]**

Solving to give $Q = \$819.98 \Rightarrow \$820$ to the nearest ten **[A1]**

Alternatively using the TVM solver

N=360
I%=6.5
PV=–130000
PMT (to be solved)
FV=0
P/Y=12
C/Y=12

PMT solves to give $\$821.69 \Rightarrow \$820$ to the nearest ten

(the answer differs slightly from the formula as that method used a rounded value of $R$)

**c** Using the TVM solver

N (to be solved)
I%=6.5
PV=–130000
PMT =900
FV =0
P/Y=12
C/Y=12 **[M1]**

N solves to give 282.32 monthly payments therefore he will make 283 payments in total. He will therefore save 77 payments. **[A1]**

## Question 3

**a** $24 \times 125 - 2400 = \$600$ **[A1]**

**b** Using the formula

$$r_e = r_f \times \frac{2n}{n+1}$$

$$r_e = 12.5 \times \frac{2 \times 24}{24+1}$$ **[M1]**

$r_e = 24\%$ per annum **[A1]**

**Total: 15 marks**

# Module 5: Networks and decision mathematics

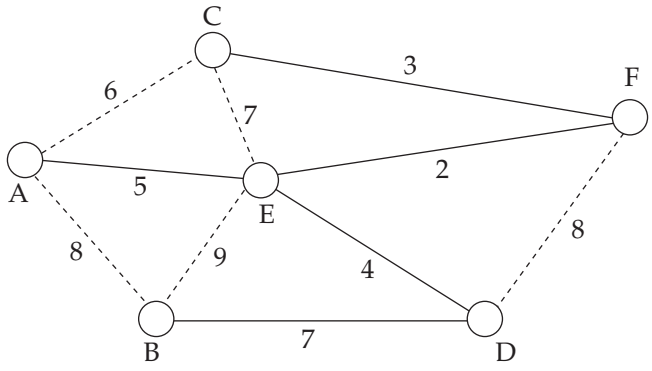
## Question 1

**a** There are a number of possible routes. One typical answer being

$D \to B \to A \to C \to F \to E \to D$ **[A1]**

**b** A Hamiltonian circuit **[A1]**

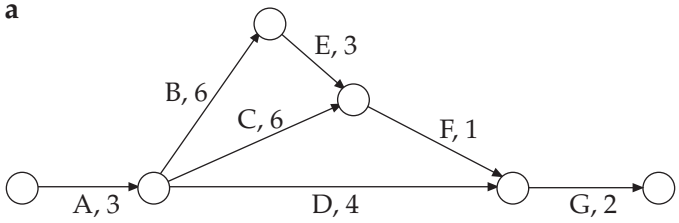
**c** **[A1]**




**d** 2+3+4+5+7 = 21 km **[A1]**

## Question 2

**a**




E correct **[M1]**

F correct **[M1]**

(arrows must be included)

**b** Earliest start time for F is 12 **[A1]**

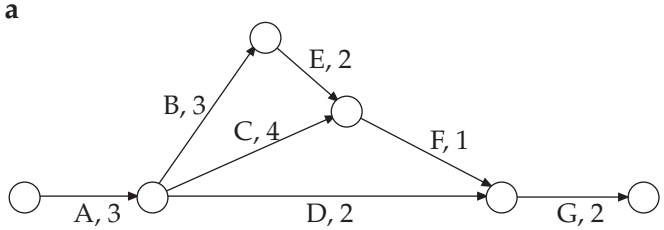
Latest start time for C is 6 **[A1]**

**c** Critical path $A \rightarrow B \rightarrow E \rightarrow F \rightarrow G$ **[A1]**

**d** 3+6+3+1+2 = 15 weeks **[A1]**

## Question 3

**a**




Drawing up new network **[M1]**

3+3+2+1+2 = 11 weeks **[A1]**

**b** Cost of reducing by 3 weeks = \$12 000 **[A1]**

This is achieved by reducing E by 1 week and B by 2 weeks

i.e. $1 \times 2000 + 2 \times 5000$ **[M1]**

Activities to be reduced to save 4 weeks are E, B and C **[A1]**

B may be reduced by a further week to 3 but this would not be beneficial unless C is also reduced by 1 week otherwise C would take 1 more week than the combined totals of B and E.

Cost of reducing by 4 weeks = \$16 000

i.e. $1 \times 2000 + 2 \times 5000 + 1 \times 4000$

**Total: 15 marks**