MAV Further Mathematics Examination 1
Answers & Solutions

Answers (Multiple-choice)

SECTION A

Core : Data analysis

1. **B**	2. **C**	3. **E**	4. **D**	5. **E**
6. **D**	7. **C**	8. **A**	9. **E**	10. **C**
11. **A**	12. **D**	13. **A**		

SECTION B

Module 1: Number patterns and applications

1. **C**	2. **D**	3. **A**	4. **D**	5. **E**
6. **D**	7. **C**	8. **B**	9. **B**	

Module 2: Geometry and trigonometry

1. **D**	2. **A**	3. **A**	4. **C**	5. **B**
6. **D**	7. **E**	8. **D**	9. **A**	

Module 3: Graphs and relations

1. **E**	2. **A**	3. **B**	4. **E**	5. **C**
6. **C**	7. **B**	8. **C**	9. **D**	

Module 4: Business related mathematics

1. **E**	2. **C**	3. **A**	4. **B**	5. **B**
6. **D**	7. **A**	8. **B**	9. **D**	

Module 5: Networks and decision mathematics

1. **D**	2. **B**	3. **C**	4. **B**	5. **D**
6. **B**	7. **A**	8. **A**	9. **D**	

Solutions

Core : Data analysis

1. **[B]**

The data is stretched at the upper end so it is best described as positively skewed.

2. **[C]**

There are 50 men so the median is the average of the 25^{th} and 26^{th} data values in the ordered set. These are both 20 so the median is 20.

3. **[E]**

The mean and the standard deviation would both be affected by a change in data because their calculation involves summing the data.

The range would be affected because 45 is the maximum value.

Inserting another 25 in the data set changes the upper quartile from 26 to 25 and hence the inter-quartile range is affected.

The median remains unchanged.

4. **[D]**

An ***r*** value of -0.7 indicates moderate, **negative** correlation, hence C or D.

D shows the 'scattering' of points associated with moderate correlation; C would be 'strong'.

5. **[E]**

"At least one" means *one* or *more than one* sport played outside school.

18 females play one sport and 3 play more than one; a total of 21 out of 44 females.

$$\frac{21}{44} \times \frac{100}{1} \approx 48\%$$

6. **[D]**

A category is chosen for each of the variables hence they are both categorical variables.

© The Mathematical Association of Victoria

7. **[C]**

A: not always true; if $r^2 = 0.35$ then
$r = \pm 0.59$; moderate.

B: Most probably 'As the temperature increases the number of pies decreases'

C: is the correct statement for an r^2 value of 0.35

8. **[A]**

Average number of marks taken
$= -7.90 + 0.065 \times 188$
$= 4.32$

9. **[E]**

The predicted 'average number of marks per football match' for a height of 181 cm
$= -7.9 + 0.065 \times 181$
$= 3.865$

Residual = Actual – Predicted
$= 3.16 - 3.865$
$= -0.705$

10. **[C]**

There is no apparent seasonality to the data but there is an obvious upward trend.

11. **[A]**

There are 10 data points so divide them 3 : 4 : 3

The three median points are approximately (2, 340), (5.5, 390) and (9, 690)

Placing a ruler on the two outer points and moving 1/3 of the way towards the middle point gives the 3-median line regression line.

The gradient is 50 and the intercept approx. 200.

12. **[D]**

The sum of the seasonal indices will be 4 for 4-season data.

SI for Spring $= 4 - (0.87 + 0.94 + 1.12)$
$= 1.07$

13. **[A]**

Deasonalised figures are found by dividing the 'Actual' by the 'Seasonal Index'.

Deseasonalised figures for *Autumn*

$= \frac{\text{Actual}}{0.94}$ and so will increase because you are dividing by a number less than 1.

Deseasonalised figures for *Winter*

$= \frac{\text{Actual}}{1.12}$ and so will decrease because you are dividing by a number greater than 1.

Module 1 : Number patterns and applications

1. **[C]**

There are total of 10 shares (5 + 3 + 2 = 10) so each share is worth \$1 000.

Peter	:	Paul	:	Mary
5	:	3	:	2
\$5 000	:	\$3 000	:	\$2 000

Therefore Paul has \$2 000 less than Peter.

2. **[D]**

2 cm : 1 km or = 1 cm : 500 m
= 1 cm : 50 000 cm
= 1 : 50 000

3. **[A]**

Total volume of cordial drink is
250ml + 2250ml = 2500ml

Strength of cordial drink = $\frac{250 \text{ ml}}{2500 \text{ ml}} \times \frac{100}{1} = 10\%$

4. **[D]**

10, 16, 22, 28…....Arithmetic, $a = 10$, $d = +6$

$S_{15} = \frac{15}{2}[2 \times 10 + (15 - 1) \times 6]$
$= 780$

© The Mathematical Association of Victoria

5. **[E]**

Geometric Mean =

6^{th} term = $\sqrt{160 \times 10} = \sqrt{1600} = 40$

Ratio = $\frac{t_6}{t_5} = \frac{40}{160} = \frac{t_7}{t_6} = \frac{10}{40} = \frac{1}{4}$

$t_5 = ar^{n-1}$

$160 = a\left(\frac{1}{4}\right)^4$

$160 = a \times \frac{1}{256}$

$a = 160 \times 256 = 40960$

6. **[D]**

The sequence is 16, –8, 4, –2, 1,…… is a geometric sequence where

Geometric ratio = $\frac{t_2}{t_1} = \frac{-8}{16} = \frac{t_3}{t_2} = \frac{4}{-8} = -\frac{1}{2}$

therefore a negative geometric sequence.

7. **[C]**

Geometric, $a = 5$, $r = \frac{1}{4}$ $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{5}{1-\frac{1}{4}}$ $S_\infty = \frac{5}{\frac{3}{4}}$

$S_\infty = 5 \times \frac{4}{3} = \frac{20}{3} = 6\frac{2}{3} \approx 6.67$ mm

8. **[B]**

For t_4, $n = 4$

$t_n = 4(5)^{n-1} + 3 = 4(5)^{4-1} + 3 = 4 \times 125 + 3 = 503.$

9. **[B]**

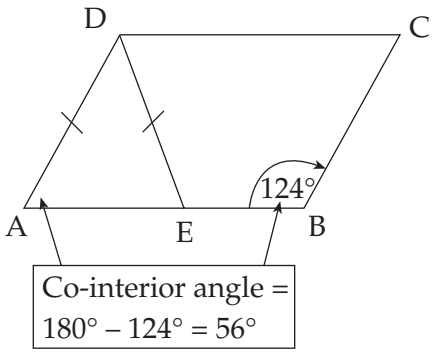
$P_n = 1.1P_{n-1} - 800$

Factor of 1.1 is an increase by 10%

Common difference of –800 or a reduction of 800 such as due to annual deaths.

Module 2 : Geometry and trigonometry

1. **[D]**



∠DEA = 56° : isosceles triangle ADE

Hence ∠ADE = 180° – 2 × 56°

= 68°

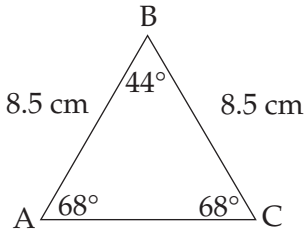
2. **[A]**

Using the cosine rule to find the length of YZ :

$YZ^2 = 320^2 + 278^2 - 2 \times 320 \times 278 \cos 37°$

YZ ≈ 193.88

3. **[A]**

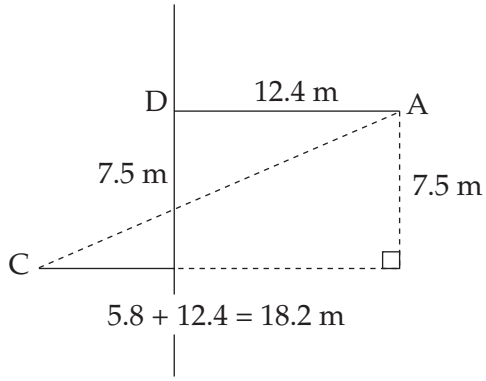


Area of triangle = $\frac{1}{2}ac \sin B$

$= \frac{1}{2} \times 8.5^2 \times \sin 44°$

≈ 25.09 cm^2

4. **[C]**

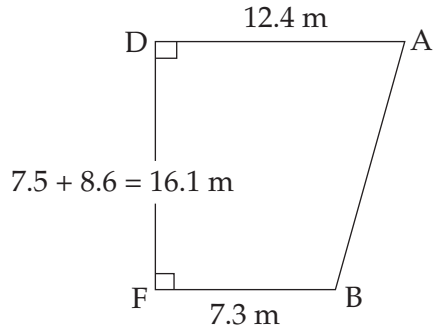


$CA^2 = 7.5^2 + 18.2^2$

CA ≈ 19.68m

© The Mathematical Association of Victoria

5. **[B]**



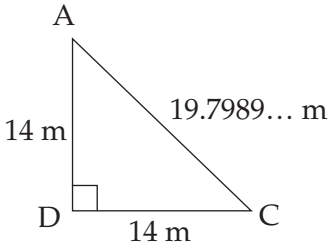
Trapezium DABF

$$\text{Area} = \frac{1}{2} \times 16.1 \times (12.4 + 7.3)$$
$$\approx 158.59 \text{ m}^2$$

6. **[D]**

Find the length of the diagonal of the base first : $AC^2 = 14^2 + 14^2$

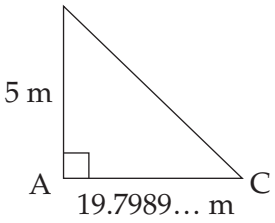
$AC = 19.7989...$m



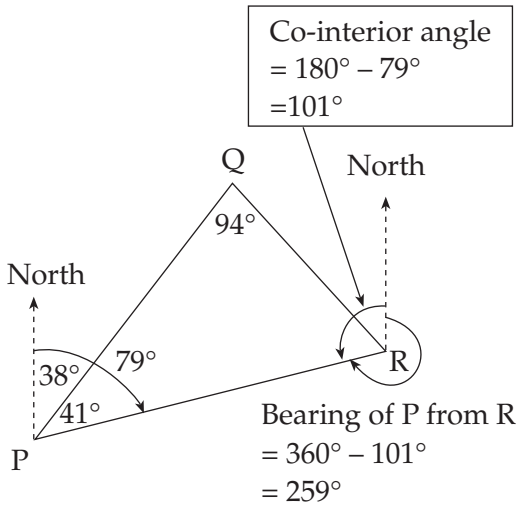
The length of the sloping edge : V

$VC^2 = 5^2 + 19.7989...^2$

$VC \approx 20.4$m



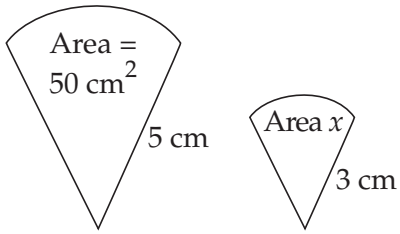
7. **[E]**



Bearing of P from R
= 360° – 101°
= 259°

8. **[D]**

Similar figures:



Length ratio 5 : 3

Area ratio $25 : 9 = 50 : x$

$x = 18$

Shaded area $= 50 - 18$

$= 32 \text{ cm}^2$

9. **[A]**

$2.4 \times 10000 = 24000$cm

$= 240$ m

$$\text{Average slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{100 - 60}{240}$$
$$= \frac{40}{240}$$
$$= \frac{1}{6}$$

© The Mathematical Association of Victoria

Module 3 : Graphs and relations

1. **[E]**

A graph of the form $y = kx^3$ has the shape where it exists in quadrants 2 and 4.

2. **[A]**

This is the only pair of equations with different gradients. All of the others are pairs of parallel lines with the same gradient.

3. **[B]**

The straight-line rule is of the form

$y = mx + c$ where $m = \frac{\text{rise}}{\text{run}} = \frac{3}{7}$ and a y-intercept of +4.

Gives $y = \frac{3}{7}x + 4$ or transposed to give

$7y - 3x = 28$

4. **[E]**

Straight line with a positive gradient represents a constant rate of change i.e. constant speed.

A horizontal line i.e. gradient = 0 represents no change in distance i.e. stopped

A upward curved line represents an increasing gradient i.e. increasing speed.

5. **[A]**

A required region above a solid line represents **greater than or equal to** (≥).

The equation of the line in the form of

$y = mx + c$

From the graph $c = 4$ and $m = \frac{\text{rise}}{\text{run}} = \frac{4}{8} = \frac{1}{2}$

so the rule is $y = \frac{1}{2}x + 4$ and rearranged with the greater than or equal to sign becomes

$2y - x \geq 8$

6. **[C]**

Solving simultaneously using elimination technique

$$\begin{aligned} x - y &= 2 \quad (1) \\ 4x + y &= 3 \quad (2) \\ (1) + (2) \quad 5x &= 5 \\ x &= 1 \end{aligned}$$

sub $x = 1$ in (1) or (2) to obtain $y = -1$ and so the solution is (1, –1)

7. **[B]**

For break even

$$\begin{aligned} \text{Cost (C)} &= \text{Revenue (R)} \\ 1100 + 2.4n &= 8.4n - 100. \\ 1200 &= 6n \\ n &= 200 \end{aligned}$$

8. **[C]**

The straight-line rule is of the form $y = mx + c$

where $m = \frac{\text{rise}}{\text{run}} = \frac{3}{12} = \frac{1}{4}$ and a y-intercept of $c = 0$. So the rule is $y = \frac{1}{4}x$ and as the x-axis is the variable $\frac{1}{R}$ and y-axis is I, the equation becomes $I = \frac{1}{4R}$

9. **[D]**

Using the maximizing function $Z = 22x + 35y$, and substituting the co-ordinates of the four corners

(0, 10)
$Z = 22(0) + 35(10)$
$Z = 350$

(5, 10)
$Z = 22(5) + 35(10)$
$Z = 460$

(10, 5)
$Z = 22(10) + 35(5)$
$Z = 395$

(10, 0)
$Z = 22(10) + 35(0)$
$Z = 220$

© The Mathematical Association of Victoria

Module 4 : Business related mathematics

1. **[E]**

Using the formula $A = PR^n$

$$R = 1 + \frac{r}{100} \Rightarrow R = 1 + \frac{5.2}{\frac{4}{100}} = 1.013$$

n = number of periods = $4 \times 2 = 8$

2. **[C]**

A 10% rise therefore \$99 = 1.1 × the previous cost

Previous cost = $\frac{\$99}{1.1} = \$90$

3. **[A]**

Value after

$$t \text{ years} = \text{purchase price} \times \left(1 - \frac{\text{depreciation rate}}{100}\right)^t$$

$$= 25000 \times \left(1 - \frac{25}{100}\right)^3$$

$$= 25000 \times (0.75)^3$$

4. **[B]**

As the account earns simple interest, the interest added at the end of each year is a constant figure

5. **[B]**

Transposing the simple interest formula

$$r = \frac{100I}{Pt} \Rightarrow r = \frac{100 \times 300}{5000 \times 2} \Rightarrow r = 3\%$$

6. **[D]**

Using the formula for effective interest

$$r_e = \frac{2I}{(n+1)P} \times m \times 100\%$$

$$= \frac{2 \times 660}{(36+1)2200} \times 12 \times 100\%$$

$$= 19.45\%$$

7. **[A]**

Minimum balance = \$780

3% of \$780 = \$23.40 per annum

$\$23.40 \div 12$ = \$1.95 per month

8. **[B]**

Using the formula $A = PR^n$ where

$$R = 1 + \frac{r}{100}$$

$$20000 = P \times 1.065^3$$

$$P = \$16556.98$$

9. **[D]**

Decreasing the interest rate, increasing the repayments or making an extra payment all clearly would reduce the term. Changing the timing of repayments reduces the term if payments are made more frequently as the interest is calculated on a lesser amount than previously. Therefore changing to quarterly repayments would not reduce the term of Harold's loan but , in fact, increase it.

© The Mathematical Association of Victoria

Module 5 : Networks and decision mathematics

1. **[D]**

A complete graph with n vertices has $\frac{n(n-1)}{2}$ edges.

Therefore a complete graph with 6 vertices has $\frac{6 \times 5}{2} = 15$ edges

2. **[B]**

The numbers in each matrix represent the numbers of edges connecting the two vertices in question. Two edges connect A and B and also C and D resulting in a 2 in the corresponding positions in the matrix. Similarly for those vertices with 1 edge connecting them and those that are not connected.

3. **[C]**

The definition of a **planar graph** is one that can be drawn in such a way that the edges have no intersections except at the vertices.

4. **[B]**

To identify an Euler circuit (a path that includes every edge just once and starts and finishes at the same vertex) look for a connected graph where all vertices have an even degree.

5. **[D]**

Tour E travels to the most countries therefore A is incorrect

Tour E also goes to Belgium therefore B is incorrect

Croatia may only be visited on Tour B therefore C is incorrect

D is correct

Tours D and E together offer 4 countries which is the same as Tours B and C together therefore E is incorrect

6. **[B]**

Using Euler's formula

$v - e + f = 2$

$4 - e + 3 = 2$

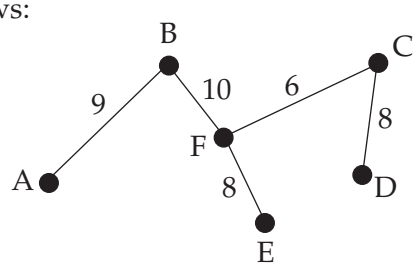
$e = 5$

7. **[A]**

A Hamilton circuit is a path through a graph that passes through each vertex only once, starting and finishing at the same vertex. A is the only option that satifies this definition

8. **[A]**

The minimum spanning tree is drawn as follows:



The weight is the sum $9 + 10 + 6 + 8 + 8 = 41$

9. **[D]**

The capacity is the sum of the flows that are from left to right across the cut

i.e. 5 + 3 + 5 =13

The 2 is not included as the flow is from right to left.

© The Mathematical Association of Victoria