Further Mathematics Examination 1
Answers & Solutions

Answers – Multiple Choice

SECTION A

Core : Data analysis

1. **C**	2. **A**	3. **D**	4. **A**	5. **E**
6. **B**	7. **B**	8. **B**	9. **A**	10. **C**
11. **C**	12. **A**	13. **C**		

SECTION B

Module 1: Number patterns and applications

1. **C**	2. **D**	3. **E**	4. **E**	5. **A**
6. **B**	7. **D**	8. **D**	9. **C**	

Module 2: Geometry and trigonometry

1. **E**	2. **D**	3. **B**	4. **C**	5. **A**
6. **A**	7. **B**	8. **C**	9. **D**	

Module 3: Graphs and relations

1. **A**	2. **E**	3. **C**	4. **A**	5. **B**
6. **A**	7. **C**	8. **C**	9. **B**	

Module 4: Business related mathematics

1. **C**	2. **D**	3. **D**	4. **A**	5. **A**
6. **B**	7. **C**	8. **E**	9. **A**	

Module 5: Networks and decision mathematics

1. **E**	2. **E**	3. **B**	4. **C**	5. **D**
6. **D**	7. **D**	8. **C**	9. **D**	

Solutions — Multiple Choice

SECTION A

Core : Data analysis

Question 1 **[C]**

Each athlete can be categorised according to their nationality.

Question 2 **[A]**

The median is the preferred measure over the mean as it is not affected by what appears to be an outlier of 41.
The mode is inappropriate as no value is repeated.
The standard deviation and the interquartile range are measures of spread.

Question 3 **[D]**

$$\text{The mean} = \frac{(0\times1)+(1\times1)+(2\times7)+(3\times3)+(4\times4)}{16}$$
$$= 2.5$$

Question 4 **[A]**

The points clearly show a weak to moderate negative association therefore –0.4 is the most appropriate alternative.

Question 5 **[E]**

95% of the heights are within 2 standard deviations of the mean i.e. between 140 cm and 180 cm. The remaining 5% is outside this range of which 2.5% will be less than 140 cm and 2.5% will be greater than 180 cm.

Question 6 **[B]**

The coefficient of determination is the square of Pearson's correlation coefficient $\Rightarrow r^2 = (-0.7)^2$
$= 0.49$
As a percentage this value gives the amount of variation in the dependent variable (performance) which can be explained by the variation in the independent variable (stress).

Question 7 **[B]**

Since the distribution 'tails off' to the left then it is said to be skewed to the left or negatively skewed.

Question 8 **[B]**

Substituting 50 into the equation

Year 10 result $= 1.835 \times 50 - 48.75$
$= 43$

Question 9 **[A]**

Substituting Angela's Year 9 result of 62 into the equation gives a Year 10 prediction of 65.02

Residual value = actual Year 10 result – predicted Year 10 result
= 60 – 65.02
= –5 (rounded off)

Question 10 **[C]**

A transformation is required that will stretch out the upper end of the x scale.
An x^2 transformation will achieve this.

Question 11 **[C]**

Since the seasonal indices must add up to 4, the two missing values must add up to 2.09, therefore the higher one of them must be at least 1.05 which will make it the highest of the four seasons.

Question 12 **[A]**

There is no clear trend upwards or downwards nor there is any evidence of a seasonal or cyclical pattern, therefore essentially random.

Question 13 **[C]**

Smoothing by averaging 2 years at a time would give a value of 3 midway between 1992 and 1993 and a value of 2 midway between 1993 and 1994. To centre, the values of 3 and 2 are averaged to give 2.5

SECTION A

Module 1 : Number patterns and applications

Question 1 **[C]**

An arithmetic sequence must have a common **difference.**

$t_2 - t_1 = t_3 - t_2 = d$

For C the common difference is 1.03

Question 2 **[D]**

The sequence is a geometric sequence.

$\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{3}{2}$ hence $r = \frac{3}{2}$ and $a = 8$

The general expression for a geometric sequence is

$$t_n = a \times r^{n-1}$$
$$= 8 \times \left(\frac{3}{2}\right)^{n-1}$$

Question 3 **[E]**

The sequence is arithmetic with

$a = -101$ and $d = 9$

$$S_{32} = \frac{32}{2}[2 \times (-101) + 31 \times 9]$$
$$= 1232$$

Question 4 **[E]**

\$57 120 is divided into 11 + 9 + 14 = 34 parts and Brad gets 14 of these parts.
\$57 120/34 = \$1680
Hence Brad gets 14 × \$1680 = \$23 520

Question 5 **[A]**

The ratio is 24 km : 7.2 cm
Dividing both sides of the ratio by 7.2 gives

$\frac{10}{3}$ km : 1cm which means that 10 km is represented by 3 cm

Question 6 **[B]**

The terms of the sequence are decreasing but not at a constant rate; they would be in a straight line if the decrease was constant, so the sequence is not arithmetic.
The difference equation, in E, generates the terms 10, 8, 6, …which is not the case.
Geometric sequences with r values between 0 and 1 have this type of graph. If $r > 1$ then the sequence is increasing and if $r < 0$ then the sequence is alternating positive/negative.

Question 7 [D]

One litre of a 10% drug mixture contains 100ml of drug (10% of 1000ml)
Substituting in the formula for strength as a percentage :

$$\text{Strength} = \frac{\text{volume of drug}}{\text{total volume}} \times 100$$

gives

$$4 = \frac{100}{\text{total volume}} \times 100$$

and solving for *total volume :*

$$\text{total volume} = \frac{100 \times 100}{4}$$
$$= 2500 \text{ ml}$$

The volume of the mixture is already 1 l so 1.5 l of water needs to be added to make up the total volume of 2.5 l

Question 8 [D]

The sequence is 2.24, 0.90, 0.36, …
(figures rounded to 2 decimal places)
A percentage decrease indicates a geometric sequence with a = 2.24 and r = 0.4

Since r < 1, $S_\infty = \frac{a}{1-r}$ exists.

$$S_\infty = \frac{2.24}{1-0.4} = \frac{2.24}{0.6} = 3.7333\ldots$$

Question 9 [C]

Substituting the value of t_2 in the difference equation to find t_1 gives :

$6 = 4t_1 - 6$
$12 = 4t_1$

Hence $t_1 = 3$

Substituting the value of t_2 in the difference equation to find t_3 gives :

$t_3 = 4 \times 6 - 6$
$t_3 = 18$

Hence $t_1 = m = 3$ and $t_3 = n = 18$

Module 2 : Geometry and trigonometry

Question 1 [E]

N42°W is 42° anti-clockwise from North or 360°T.
360° – 42°= 318° T

Question 2 [D]

This is an isosceles right-angled triangle where the lesser side is smaller than the hypotenuse by a factor of $\sqrt{2}$.

$$x \times \sqrt{2} = 10\sqrt{2}$$
$$x = 10 \text{ metres}$$

Or using Pythagoras' Theorem
$a^2 + b^2 = c^2$ and substituting, we get

$$x^2 + x^2 = \left(10\sqrt{2}\right)^2$$
$$2x^2 = 100 \times 2$$
$$x^2 = 100$$
$$x = \sqrt{100} = 10 \text{ metres}$$

Also can use Trigonometry function cosine or sine as both unknown angles are 45°.

Question 3 [B]

Using Tan function where

$\theta = \text{Tan}^{-1}\frac{4}{8} = 26.565°$ and using Alternate Angle rule the angle of decline from the cliff is closest to 27°

Question 4 [C]

35 mm is enlarged to 210 mm (21cm).

$$\text{Linear Scale Factor, LSF} = \frac{210 \text{ mm}}{35 \text{ mm}} = 6$$

$$\text{Area Scale Factor} = (\text{LSF})^2 = 6^2 = 36$$

Question 5 [A]

Using Sine rule where

a = $\overline{LM}$ A° = 44° b = 10cm B° = 65°

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{\sin 44°} = \frac{10}{\sin 65°}$$

$$a = \frac{10 \times \sin 44°}{\sin 65°} \cong 7.7 \text{ cm}$$

Question 6 [A]

Shaded Area = Rectangular Area – Triangular Area

Area of Rectangle = L × W = 6 × 2 =12 m^2

Length of base of triangle is

$$x = 2\text{ m} \times \tan 40°$$
$$= 1.8008\ldots\text{ m}$$

Area of Triangle $= \frac{1}{2} \times 1.8008 \times 2 = 1.8008\text{ m}^2$

Shaded Area = 12 – 1.8008 = 10.1992 = 10.2 m^2

Question 7 [B]

Using the cosine rule where

$$\angle B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

where $a = 5$ $b = 8$ and $c = 4$, substituted gives

$$\angle B = \cos^{-1}\left(\frac{5^2 + 4^2 - 8^2}{2 \times 5 \times 4}\right)$$

Question 8 [C]

Using cosine rule to find angle of the triangle at Mt Beauty

$$\theta = \cos^{-1}\left(\frac{29^2 + 40^2 - 60^2}{2 \times 29 \times 40}\right)$$
$$= 119.97° \cong 120°$$

The rotation from South is then 120° – 65° = 55°. Therefore the bearing is S55° E.

Question 9 [D]

First offset is 15 units to the left, 10 units from C.
Second offset is 25 units to the right, 25 from C.
Third offset is (15 + 5) units to the left, 45 from C.
Point E is 85 units from C.

Module 3 : Graphs and relations

Question 1 [A]

An equation of this form represents a vertical line, as the gradient is not defined, with an x intercept of 1.

Question 2 [E]

Rearrange into the form $y = mx + c$

$3x - 3y = 6$
$3y = 3x - 6$
$y = x - 2$
therefore gradient =1

Question 3 [C]

$$\text{Cost} = 0.45 \times \text{number of kilometres} + 3.80$$
$$12.80 = 0.45n + 3.80$$
$$0.45n = 9$$
$$n = 20$$

Question 4 [A]

$$3x - 4y = 16 \quad (1)$$
$$x + 5y = -1 \quad (2)$$

Eliminate by multiplying equation 2 by 3 and subtracting

$$3x - 4y = 16$$
$$3x - 15y = -3$$
$$-19y = 19$$
$$y = -1$$

substitute $y = -1$ in equation 2

$$x - 5 = -1$$
$$x = 4$$

Question 5 [B]

An equation of this form will occur if the relationship between y and x^2 is linear.

Question 6 [A]

The steepest section is clearly from A to B.

Question 7 [C]

The line must have a negative gradient if y decreases as x increases.

Question 8 [C]

Test each of the extreme points labelled. The maximum value is 65 at (30, 25).

Question 9 [B]

The charge is $15 for usage up to and including 10 hours. Therefore Wendy and Roger will each be charged $15 making a total of $30.

Module 4 : Business related mathematics

Question 1 [C]

Multiply each rate by the number of compounds per year.

Bank A	monthly	$\therefore 12\times 0.5 = 6.0\%$
Bank B	monthly	$\therefore 12\times 0.6 = 7.2\%$
Bank C	quarterly	$\therefore 4\times 2.0 = 8.0\%$
Bank D	biannually	$\therefore 2\times 3.8 = 7.6\%$
Bank E	annually	$\therefore 1\times 7.5 = 7.5\%$

Bank C offers the best annualized rate of 8.0%.

Question 2 [D]

$$\text{Rate of Depreciation} = \frac{\$5000}{\$30000}\times\frac{100}{1}\%$$
$$= 0.1666\ldots\times 100\%$$
$$= 16.\overline{6}\%$$
$$\cong 17\%$$

Question 3 [D]

$$\text{Depreciation after 1 year } \$ = 10\%\times\$80000$$
$$= \$8000$$

Value after 1 year = Purchase price – Depreciation \$
= \$ 80 000 – \$ 8 000
= \$ 72 000

$$\text{Depreciation after year 2 } \$ = 10\%\times\$72\,000$$
$$= \$7\,200$$

Value after 2 years = 1st year price – Depreciation \$
= \$ 72 000 – \$ 7200
= \$ 64 800

Question 4 [A]

The Balance is **growing** at a **constant rate** (or linear growth). This is **simple interest savings** account.

Question 5 [A]

Term deposit is simple interest.

$$R = \frac{100SI}{PR} = \frac{100\times 1000}{5000\times 3} = 6.66\ldots\% \cong 6.7\%$$

Question 6 [B]

P = \$1000 R = 1 + 2% per quarter = 1.02
T = 3years n = 4 x 3 = 12 periods

$$A = PR^n$$
$$= 1000\times 1.02^{12}$$

Question 7 [C]

Compounding factor = R = 1.005

$$R = 1+\frac{r}{100} \text{ where r = interest rate per period}$$
$$= 1+0.005 = 1+\frac{0.5}{100}$$
$$r = 0.5\% \text{ per month}$$
$$= 0.5\times 12 = 6.0\% \text{ per year}$$

Question 8 [E]

Loan Amount = \$ 8 200

$$\text{Interest} = \frac{PRT}{100} = \frac{8200\times 5\times 5}{100} = \$2050$$

Total Repayments = \$ 8 200 + \$ 2 050 = \$10 250

$$\text{Monthly Instalments} = \frac{\$10\,250}{60} = \$170.83$$

Question 9 [A]

Rate of Depreciation = \$2.50 per 500sheets

$$\text{Total Depreciation} = \$2.50\times\frac{1200000}{500}$$
$$= \$2.50\times 2400 \text{ reams}$$
$$= \$6000$$

Bookvalue = Previous Bookvalue – Deprecation
= \$12 500 – \$6 000
= \$6 500

Module 5 : Networks and decision mathematics

Question 1 [E]

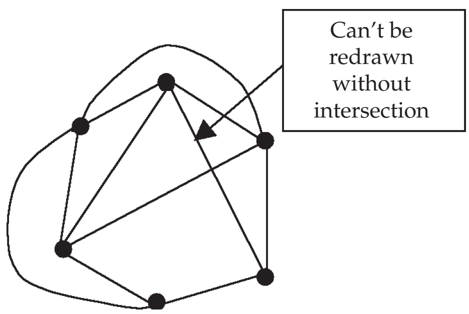
A complete graph has an edge joining every pair of nodes. (not A)
A directed graph has arrows showing direction on the edges. (not B)
A tree is a connected graph that has no circuits. (not C)
A weighted graph has a number associated with each edge. (not D)
A simple graph has no multiple edges or loops. (Hence E)

Question 2 [E]

Jess and Jason are the only two athletes who have entered the highjump.

Question 3 [B]

A planar graph is one that can be drawn so that no edges intersect.
Although two of the edges with intersections can be redrawn there will always be an intersection of edges with this graph.



Question 4 [C]

Using Euler's formula : $v - e + f = 2$

$$\begin{aligned} 7 - e + 7 &= 2 \\ 14 &= e + 2 \\ 12 &= e \end{aligned}$$

Question 5 [D]

The critical path for this project is
C E F H so the earliest and latest start times will be the same for activity H.
Activities C, E and F take 3 + 8 + 4 = 15 days so the latest time that activity H can begin is day 16.

Question 6 [D]

The sum of the degrees of the graph is even.
2 + 4 + 3 + 4 + 3 + 4 = 20 so True
A Hamiltonian path goes through each of the nodes once only not starting and finishing on the same node.
e.g. ABCDEF so True
A Euler path goes along each of the edges once only not starting and finishing on the same node.
e.g. CBAEBDFEDCF so True
An Euler circuit can only be formed if the degree of all the nodes is even. This would not be true if the path from E to F is removed.
A Hamiltonian circuit goes through each of the nodes only once, starting and finishing on the same node.
e.g. ABCFDEA so True

Question 7 [D]

The graph is a directed graph so the edges are counted in one direction only.

Question 8 [C]

A spanning tree contains no circuits (this eliminates A and E) and is connected.
B and D have an edge going from node 5 to node 6 which does not exist.

Question 9 [D]

Cut 1 has capacity 24
Cut 2 has capacity 22 (the edges 3 and 2 are not counted as the flow is from the right to the left)
Cut 3 has capacity 25 (2 not counted)
Cut 4 has capacity 27 (2 not counted)
Cut 5 has capacity 21 (6 not counted)