JOHN MONASH SCIENCE SCHOOL

VICTORIAN CERTIFICATE OF EDUCATION

2021

STUDENT NAME:

TEACHER NAME: **ALBD** **BOHL**

ALGORITHMICS (HESS)

Practice Exam 1

2021

Reading Time: 15 minutes
Writing time: 120 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	15	15	80
		Total	100

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 31 pages.
- Answer sheet for multiple-choice questions.

Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice Questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** or that **best answers** the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \quad \text{where } a > 0, b > 1, c \geq 0, d \geq 0, k > 0$$

and its solution $T(n) = \begin{cases} O(n^c) & \text{if } \log_b a < c \\ O(n^c \log n) & \text{if } \log_b a = c \\ O(n^{\log_b a}) & \text{if } \log_b a > c \end{cases}$

Question 1

Which of the following is **not** a correct abstract data type (ADT) specification for the Queue ADT?

A. enqueue: element x queue → queue

B. dequeue: queue → element

C. is empty: queue → boolean

D. create: → queue

Question 2

Amish wants to keep track of all of the people who have wronged him this year so that he can exact revenge upon them in the future. (Sukhman is going to be first for reasons that will be revealed later). He wishes to keep the people in some sort of order based on how badly they have wronged him so that the most serious wrongdoers appear first.

Which of the following abstract data types (ADTs) would be most appropriate for Amish to use to store this information?

A. List

B. Dictionary

C. Graph

D. Priority Queue

END OF SECTION A
TURN OVER

Question 3

T is a binary tree with *n* edges.

What is the maximum number of nodes that *T* contains?

A. $n-1$

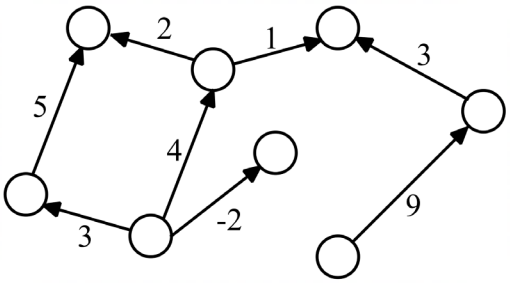
B. n

C. $n+1$

D. n^2

Question 4

Consider the following graph, *G*.



Which of the following statements about the properties of *G* are true?

A. *G* is undirected and labelled

B. *G* is directed and cyclic

C. *G* is connected and labelled

D. *G* is directed and weighted

Question 5

The proof of correctness for the Bellman-Ford algorithm involves making which of the following assumptions?

A. After *k* iterations of the outer loop, the distance from node *i* to node *j* will be the shortest distance that uses only the first *k* nodes.

B. After the k^{th} iteration of the algorithm, the distance locked in from the start node to node *i* is not the shortest distance possible.

C. After the k^{th} iteration of the algorithm, the distance from the start node to each node will the shortest distance path that contains at most *k* edges.

D. After *k* iterations, *T* is a minimal spanning tree of the $k+1$ vertices it contains

Question 6

A loop invariant is best described as:

A. a formal statement about the relationship between variables in your program which holds true just before the loop is ever run and is true again at the bottom of the loop.

B. a formal statement about the relationship between variables in your program which holds true whilst the loop is being run.

C. a formal statement about the relationship between variables in your program which is only true immediately before the loop is run for the first time.

D. a formal statement about the relationship between variables in your program which holds true just before the loop is ever run, whilst the loop is running and at the bottom of the loop.

Question 7

An algorithm has a recurrence relation given by:

$$T(n) = 3n + 2T\left(\frac{n}{4}\right).$$

Which of the following represents the Big-O time complexity of this algorithm?

A. $O\left(\sqrt{n}\right)$

B. $O\left(n\log(n)\right)$

C. $O\left(n^{\log_4(2)}\right)$

D. $O\left(n\right)$

Question 8

Yisen is investigating the differences between Mergesort and Quicksort. He compiles the following statements about the algorithms:

I Mergesort and Quicksort have the same best case asymptotic time complexity

II Quicksort has a better worst case asymptotic time complexity than Mergesort

III Mergesort is a more stable sorting algorithm than Quicksort

IV Mergesort has the same best case and worst case asymptotic time complexity

Which group of statements is correct?

A. I and II

B. I, III and IV

C. I and IV

D. II, III and IV

SECTION A - continued

Question 9

Nishant is searching for the answers to the 2021 Algorithmics Exam when he comes across the following algorithm.

```
//input: an array of values, A[0…n-1] and a value to search for
//output: an integer value corresponding to the location of x in the array

Search_Algorithm(A[0…n-1],x)
     If list is empty
          Return "Value not found"
     i = 0
     while i < n
          if A[i] = x
               return i
          else
               i = i + 1
     return "Value not found"
```

The worst-case time complexity of this algorithm is:

A. $O(1)$

B. $O(\log(n))$

C. $O(n)$

D. $O(n^2)$

Question 10

Which of the following statements is true for a connected graph?

A. Prim's algorithm always produces a unique minimal spanning tree.

B. If there is a cycle in the weighted graph, then Prim's algorithm will not work.

C. If there is a cycle in the weighted graph, then Prim's algorithm will not produce a unique spanning tree.

D. If the graph contains no cycles, then Prim's algorithm will produce a unique spanning tree.

Question 11

Consider the following pseudocode for the algorithm **Unknown**

```
Unknown(A[1,..,2n])
create an empty stack, S
create an empty queue, Q

For k ← 1 to n
    S.push(A[2*k])
    S.push(A[2*k-1])

While S is not empty
    Q.append(S.peek())
    S.pop()

While Q is not empty
    print(Q.peek())
    Q.serve()
```

Given the input A = ['a ', 'b ', 'c ', 'd ', 'e ', 'f '], what would be the output of the above algorithm?

A. abcdef

B. fedcbc

C. efcdab

D. badcfe

Question 12

Which of the following statements is true?

A. The Depth First Search algorithm can be implemented using a queue.

B. The Breadth First Search algorithm can be implemented using a queue.

C. The Depth First Search algorithm can be implemented using either a queue or a stack.

D. The Breadth First Search algorithm can be implemented using either a queue or a stack.

Question 13

Which of the following statements is true

A. Dijkstra's algorithm always finds the shortest paths between all pairs of nodes when there are no negative edge weights.

B. The Bellman-Ford algorithm will find the shortest path between two nodes when there is no negative cycle.

C. The Bellman-Ford algorithm will always find the shortest path between all pairs of nodes when there are negative edge weights.

D. Dijkstra's algorithm will find the shortest path between all pairs of nodes when there is no negative cycle.

Question 14

Which of the following statements is true about the John Searle's Chinese Room argument:

A. The Chinese Room Argument proves that no computer can understand a natural language.

B. The Chinese Room Argument provides reasons why strong AI is possible.

C. The Chinese Room Argument demonstrates that only weak AI is possible.

D. The Chinese Room Argument proves that the Turing Test is the best test for AI.

Question 15

Which of the following statements is false.

A. The Halting problem shows that there are some Turing Machines that will never halt.

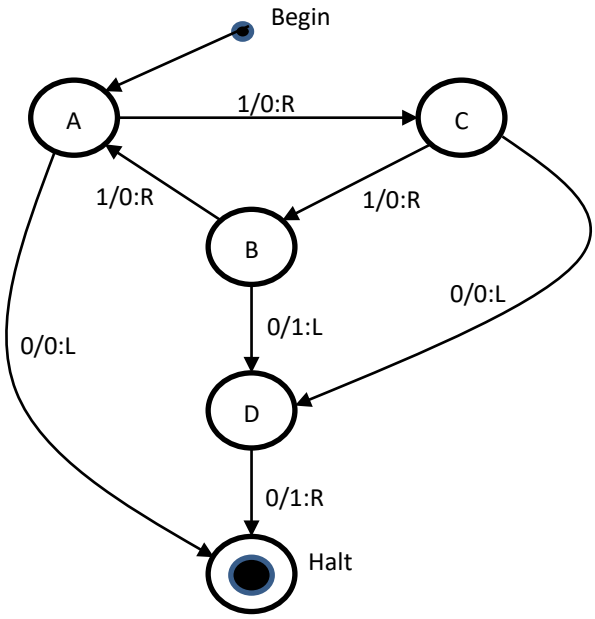
B. The Halting problem shows that we cannot tell for some Turing Machines whether they will ever halt on some input.

C. The Halting problem shows that some bugs in programs cannot be detected automatically.

D. The Halting problem shows that there are undecidable problems.

Use the following information to answer Questions 16 and 17

A Turing Machine is configured with the instructions represented in the state diagram below.



Each edge is labelled *i* / *j* : *k*, where:

- *i* is the input
- *j* is the output
- *k* is the direction the head moves (L = left, R = right) after the output is put on the tape.

The machine starts in state A.

The machine is given the following tape. For this machine, the tape remains stationary while the head moves. The arrow shows the starting point of the head.

0	0	0	1	1	1	1	1	0	0

Question 16

The Turing Machine is run for 4 steps. In what state will the Turing Machine be in at this time?

A. State B

B. State C

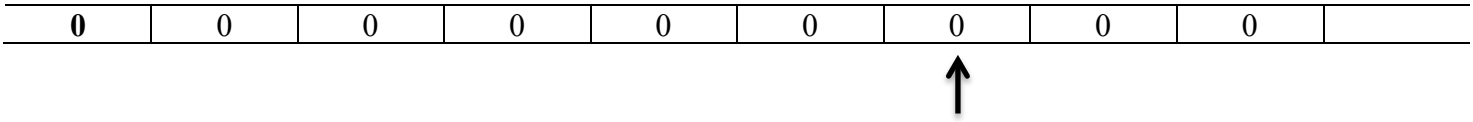
C. State D

D. State A

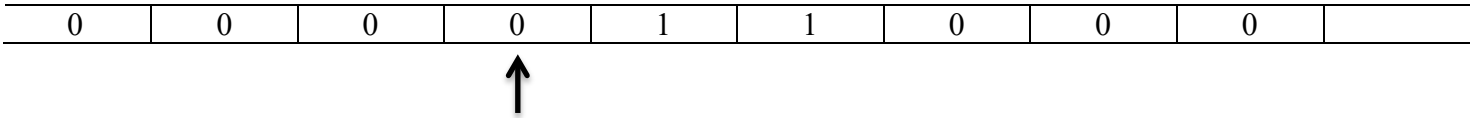
Question 17

Which one of the following best represents the tapes appearance and the position of the head when the Turing Machine halts?

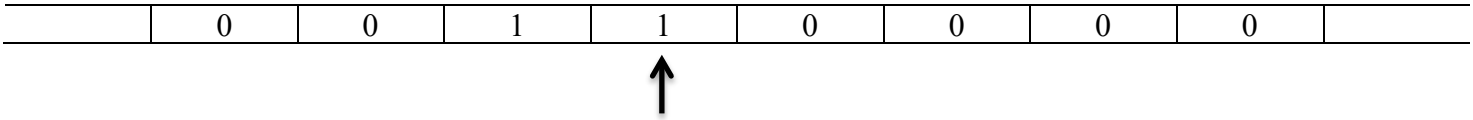
A.



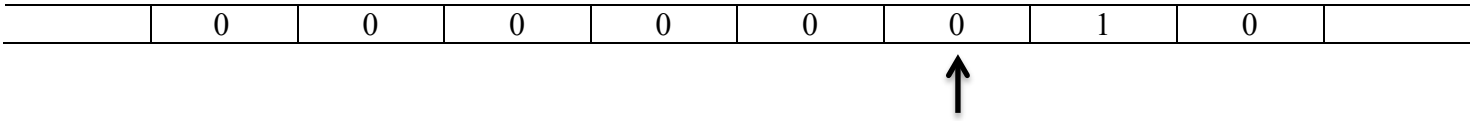
B.



C.



D.



Question 18

Which of the following statements is false

A. The Travelling Salesman Problem is important as it is used to solve many scheduling problems.

B. Finding a polynomial time algorithm that solves the Travelling Salesman Problem would have major implications on network security.

C. The Travelling Salesman Problem is important as it can only be solved using Randomized Heuristics.

D. It is not possible to always verify solutions to the Travelling Salesman Problem in polynomial time.

Question 19

Dynamic Programming is an algorithmic approach that is best used in which of the following situations?

A. When the optimal solution to a problem cannot be verified in polynomial time.

B. When the solution to a problem involves a 2-Dimensional array.

C. When the solution to a problem involves repeatedly finding the solutions to overlapping subproblems.

D. When the solution to a problem can be found by solving a version of the problem with smaller input size first.

Question 20

Mr Bohni and Dr Albrecht are having some fun messing with the year 10s. They are writing down conditional expressions and asking the year 10s to determine which ones will always be true regardless of the values of X, Y and Z.

Which of the following conditional expressions is always true regardless of the values of X, Y and Z?

A. **(**(X or Y) and (X or Z)) and (Y and Z)

B. **(**(X and Y) or (X' or Z')) or (Y or Z)

C. **(**(X or Y') and (Y or Z)) and (X or Z')

D. **(**(X or Y) or (X or Z)) and (Y and Z')

This page dislikes Algorithmics.

Algorithmics, however, likes this page.

It is a bit of an awkward situation.

That's why this page is blank.

SECTION B

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided

Question 1 (4 marks)

Darcy is fed up with PyNode and wants to write his own version of the program so that he can usurp Alex Socha and become immortalised in the JMSS Algorithmics Hall of Fame.

Write a complete signature specification for the Graph ADT.

Question 2 (5 marks)

Mr Corkill is planning a special event in 2022 for those students at JMSS who have missed their formals in 2021 due to lockdown. Students will be allowed to bring one additional person to the event but Mr Corkill needs a way to keep track of how many tickets each student is purchasing.

He decides to ask Valerii for assistance.

a. Name an appropriate ADT that Valerii could use and how it would model the relevant information for Mr Corkill. 3 marks

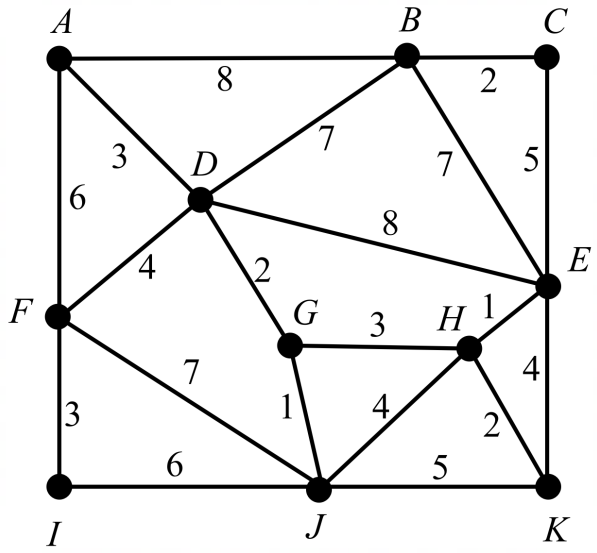
Mr Corkill realises that he will also need to keep track of whether a student has paid for the tickets and who will be picking up the tickets.

b. Explain what changes Valerii will need to make to the ADT chosen in **part a.** in order to be able to keep track of the extra information. 2 marks

Question 3 (8 marks)

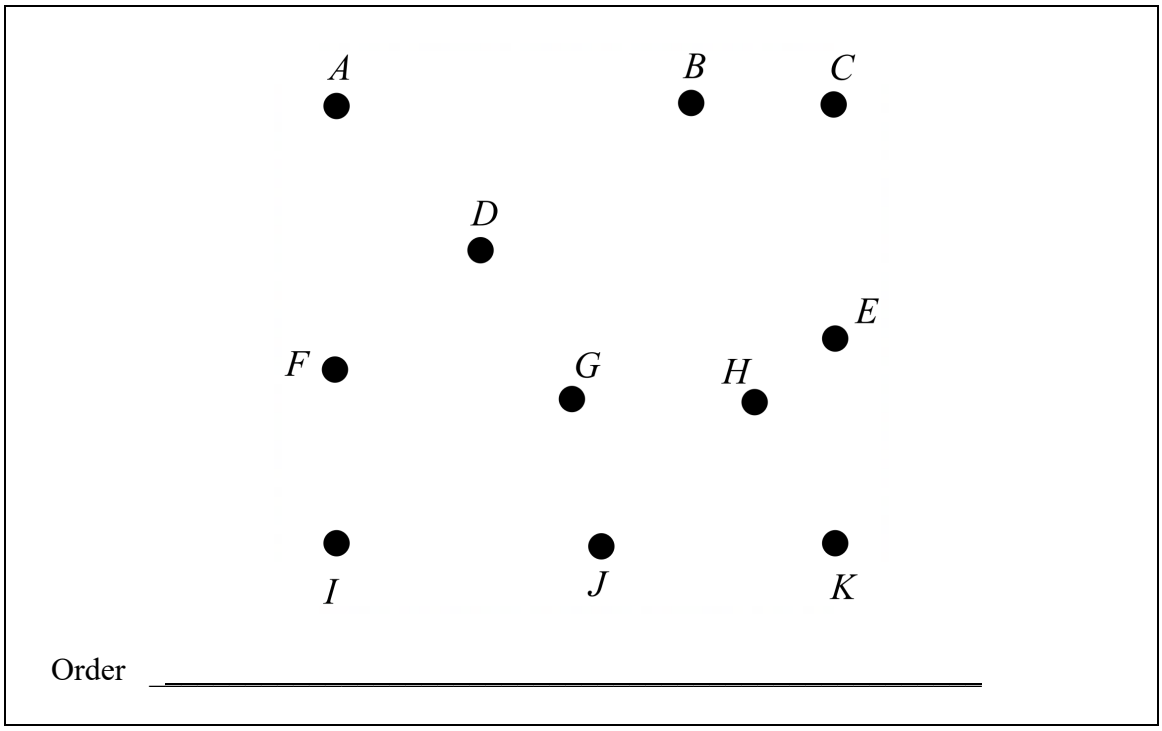
Sukhman and Jordan spent a large part of the school holidays thinking about their futures and what they wanted to do after school. Whilst nothing has been set in stone as of yet, they are pretty sure that they want to get into the construction industry, specifically, building high-tech 'smart barns'.

Their first design (shown below) has a series of 'smart sensors' placed at various locations around the barn floor. Each of these sensors is connected to the network via ethernet cables. In the diagram below, sensors are represented by nodes and ethernet cables are represented by the edges connecting them. The weights on the edges indicate the total length, in metres, of ethernet cable required for that section.



Jordan thinks that they are using too much ethernet cable and wants to remove some of the connections from the design.

a. Using Prim's algorithm, find the minimal spanning tree for the weighted graph shown above starting from *A*. Show the order of the edges added to the tree. 2 marks



SECTION B - continued

Sukhman is not convinced and argues that the minimal spanning tree network proposed by Jordan would produce too much lag (in reality the lag would actually be caused by Sukhman listening to too much Donda).

If the lag experienced between two sensors is directly proportional to the length of ethernet cable the signal has to traverse between the two sensors then:

b. Explain, with use of an example, how the minimal spanning tree does not necessarily produce the shortest path distance between two pairs of nodes. 2 marks

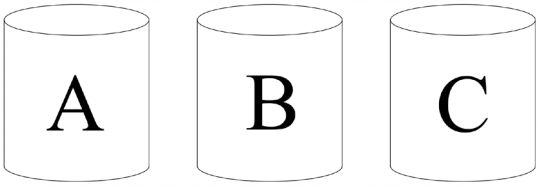
Not willing to let Sukhman discredit him so easily, Jordan wants to determine exactly what the expected lag time is between each pair of nodes in the original design. He is choosing between using Dijkstra's algorithm, the Bellman-Ford algorithm or the Floyd-Warshall algorithm.

c. For each of the named algorithms above, compare their suitability for finding the solution to the given problem and state which algorithm Jordan should use providing a justification of why it is the most suitable. 4 marks

Amish recently approached Sukhman to see if he could join in with Sukhman's and Jordan's Barn Building enterprise as he loves building barns (he is Amish after all). When he asked about it, Sukhman told him that they weren't currently looking for investors. Amish felt this was just rude and is now seriously considering becoming mortal enemies with Sukhman.

Question 4 (7 marks)

To entertain her friends Samantha has come up with a new game. It consists of three empty cans of Sustagen that have been provided to her by Mr Kermond. She labels the outside of each can with either an A, B or C.



Inside one of the cans Samantha puts a Mars bar, while she leaves the other two cans empty. She then asks Jackie to pick which can they think the Mars bar is in. Samantha knows where the Mars bar is so she then chooses one of the cans which is empty and was not picked by Jackie, and shows Jackie it is empty. Jackie can then decide whether or not to change their choice can they think the Mars bar is in.

a. Assuming that Samantha has put the Mars bas in the can labelled A, draw a decision tree to help Samantha work our which empty can she should choose to show Jackie. 3 marks

Samantha's game becomes a huge hit with Jackie and he now wants to play it all the time! Samantha would rather be spending time studying for the Algorithmics exam though so she decides to try and write an algorithm that will automatically choose which can should be revealed to Jackie each time.

b. Write pseudocode that takes as input the can with the Mars Bar in it and the can that Jackie chooses and outputs the can that Samantha should choose to show Jackie. 4 marks

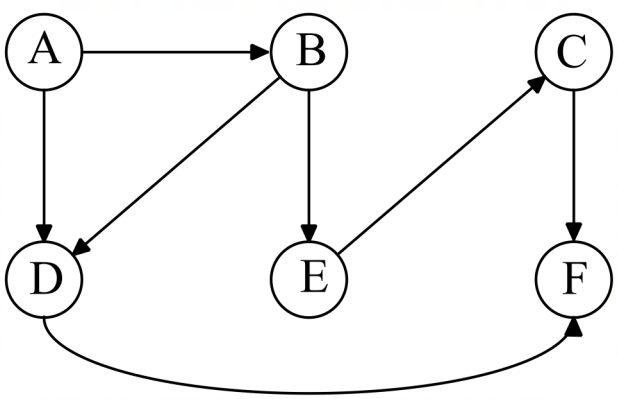
Question 5 (4 marks)

Use induction to prove that $n! > 2^n$, for $n \geq 4$.

Question 6 (2 marks)

Ainsley is looking for the next lettered mug to bring to his online Maths Methods class. For some reason, the mugs have been hidden in a labyrinth where each mug has been placed in its own room and each room can only be accessed through the doorways connecting them

The diagram below shows a portion of the labyrinth where each room is represented by a node, the label on the node represents the letter on the lettered mug and the directed edges indicate which room can be travelled to from which other room.



Assume that Ainsley starts at Node A and always chooses to travel to rooms in alphabetical order when presented with a choice between rooms.

a. Write the order in which Ainsley will visit the nodes in the labyrinth if he uses a depth first search to look for the next mug. 1 mark

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b. Write the order in which Ainsley will visit the nodes in the labyrinth if he uses a breadth first search to look for the next mug. 1 mark

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Question 7 (7 marks)

Ivan and Darren have become obsessed with a new AR game that is soaking up time that they would otherwise be spending on Algorithmics. They know that there is a portal for the game located somewhere on the ground floor of the school. In order to find the portal they have represented the ground floor as a 10x20 grid of squares. Their plan is to visit each square in the grid one-by-one, search that square for the portal using the AR game on their phones, and stop when they have found the portal.

a. Assume every square has a coordinate (x, y), where $1 \leq x \leq 10$, $1 \leq y \leq 20$, and Darren and Ivan after searching one square will go to a neighbouring square. Write some pseudocode that prints out the coordinates of the squares in the order Ivan and Darren should visit them.

3 marks

Prabhu notices what Ivan and Darren are doing and suggests that they represent the problem as a graph instead where nodes represent each square to be searched and adjacent squares have edges connecting them. Prabhu argues that this way they can use a Depth First Search to try and find the portal and thus be 'studying' Algorithmics at the same time as playing the game.

Prabhu supplies Ivan and Darren with the following pseudocode for a Depth First Search.

```
DFS(G, x, target)
Visited(x) = True
If x = target
     Return x
Else
     For each neighbour, n, of x.
          If visted(n) = False
               Return DFS(G,n)
```

b. State and give reasons for what would be the worst-case asymptotic time complexity for Prabhu's algorithm. 2 marks

c. State and give reasons for what would be the best-case asymptotic time complexity for Prabhu's algorithm. 2 marks

Question 8 (3 marks)

In her spare time Julee has developed a new way to mix up lists. In order to explain her method to Yasmin, Julee has written the following pseudocode.

```
//Input: A list L with n items
//Output: A new, mixed up List L, with n items

Algorithm mix(L,n)
    If n > 1
       Let a = the list consisting of every second item
               starting with the first item.
       Let b = the list of items in L that are not in a.

       Let a = mix(a,n/2)
       Let b = mix(b,n/2)

       Let L = the list a joined to the end of the list b.
     return L
```

Having shared her pseudocode with Yasmin, Julee then asks Yasmin to find out the time complexity of her algorithm for her since she is busy studying for her exams and can't be bothered doing it herself. Yasmin, surprisingly, agrees to help her out.

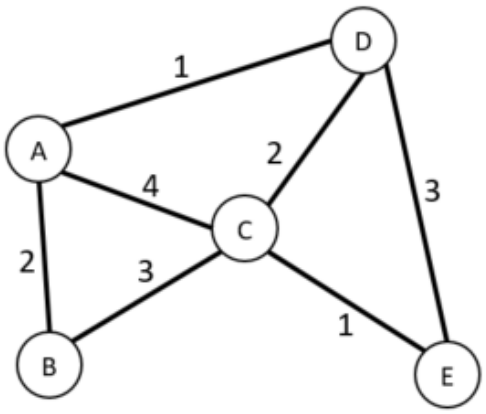
Assume that the number of operations to construct a is n/2 and b is n/2, where n is the number of items in L, and there is one more operation required to join a to the end of b.

a. Write a recurrence relation for the number of operations **mix**() will perform. 2 marks

b. What is the Big-O solution to the recurrence relation found in part **a**.? 1 mark

Question 9 (5 marks)

Consider the following graph that Andrew Nie drew one day whilst he was philosophically pondering the meaning behind the existence of the Durian.



a. Identify two features of the above graph. 2 marks

b. Describe what an Eulerian Circuit is and explain why the above graph does not contain one. 2 marks

c. Find the width of the above graph. 1 mark

Question 10 (7 marks)

Ruby has been thinking a lot about the following version of the Travelling Salesman problem during the school holidays:

"Given a set of n cities each connected by to one another by paths of set lengths, find the shortest path that will allow a travelling salesman to visit each city exactly once and return to the starting city."

She has a dream one night where she is able to find the optimal solution to this problem using Prim's Algorithm. When Ruby wakes up, she wonders whether Prim's algorithm could actually be used to find a solution in polynomial time.

a. Explain what the significance of Ruby finding a way to solve the given version of the Travelling Salesman problem in Polynomial time would be to the P vs NP question would be. 3 marks

Jeremy hears about Ruby's idea and thinks that it is unlikely that Prim's could be used to solve the Travelling Salesman Problem in polynomial time. He instead thinks that the use of a Randomised Heuristic would be more appropriate.

b. Explain how a Randomised Heuristic could be used to help find a solution to the Travelling Salesman problem and discuss the quality of this given solution compared with a brute force approach. 4 marks

Question 11 (4 marks)

Let G be a connected graph with n vertices and n-1 edges. Prove that G must have a vertex of degree 1.

Question 12 (5 marks)

Dechlan and Michael have successfully reached the highest of heights, playing against one another in the grand final of the inaugural JMSS Connect-4 competition.

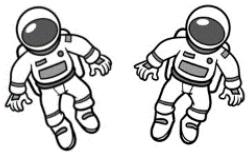
Connect-4 is a solved game, which means that an outcome can be predicted from any state of the game with the assumption that both players play perfectly.

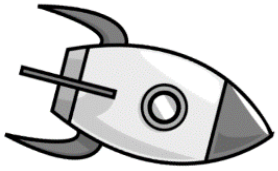
In this game, each player takes turns in placing one of their counters into one of seven vertical column of their choosing. The counter, when put into a column will occupy the space that is the lowest unoccupied state in this column unless the column is already full (a column can hold at most six counters). If they can achieve getting 4 of their counters lined up in a row either vertically, horizontally or along the diagonal without any opponent counters in the way, they will be declared the winner.

Dechlan wishes to use the Minimax algorithm to assist him in playing the game.

In the space below, describe the way in which Minimax could be used to assist Dechlan in choosing what moves he should make. As part of your answer discuss the feasibility of using an approach like this from the very beginning of the game and what assumptions the algorithm will make.

Question 13 (8 marks)





Somewhere deep in space, Aabshaar, Ben, Charlie and Dasindu are astronauts returning from a secret space mission. Unluckily for them, the computer AI that controls their spaceship has gone crazy and is refusing to open the pod-bay doors between the spaceship it controls and the pod containing the astronauts. Since they cannot dock the pod with the spaceship, the astronauts will be forced to take a difficult spacewalk between the pod and the emergency airlock on the spaceship. Fortunately each astronaut has their own space-suit.

Unfortunately they only have one portable oxygen-pack between them, which the suits require to work. This pack can be used simultaneously (if necessary) by two astronauts travelling together, but it cannot provide oxygen for three or more at the same time.

Since there is only one pack it will have to be taken back and forth until all the astronauts are safely onboard. It is a difficult space walk and abilities of the astronauts vary, so their times to travel between the pod and the space station also vary. Two astronauts travelling together move at the speed of the slowest astronaut, since they are kept together by the oxygen-pack.

For example, suppose Aabshaar (A) takes 1 minute, Ben (B) takes 2 minutes, Charlie (C) takes 4 minutes and Dasindu (D) takes 5 minutes. The following combination of walks gets the astronauts onto the space station in 13 minutes. A and D travel to the station (5 minutes), A travels back to the pod (1 minute), A and C travel to the station (4 minutes), A travels back to the pod (1 minute) and finally A and B travel to the station (2 minutes). Note that this is not the optimal solution.

a. One approach to finding a combination of walks that will get Aabshaar, Ben, Charlie and Dasindu into the spaceship as quickly as possible is to use a greedy algorithm.

Explain whether or not this is a feasible approach. Include a discussion of the optimality of the resultant solution as part of your answer. 2 marks

Unfortunately, Aabshaar, Ben, Charlie and Dasindu have no idea how to write a greedy algorithm let alone solve the given problem and so they ask a passing alien (named Sharan) for some help. Sharan is a brutish sort of alien and only like brute-force approaches though.

b. Write pseudocode for a brute force approach to finding the minimum possible time required get Aabshaar, Ben, Charlie and Dasindu into the spaceship. 4 marks

Sharan, after helping the astronauts successfully get into their spaceship, becomes very interested in the troublesome computer's Artificial Intelligence (AI) system. He and Aabshaar enter into a philosophical discussion about whether this AI system is an example of strong or weak AI.

c. In the space below, describe what John Searle's Chinese Room Argument would suggest about the nature of the spaceship's AI. 2 marks

Question 14 (5 marks)

Kody and Shiann have broken into the JMSS labs in the hope of being able to steal some gel electrophoresis kits. They want to try and use DNA computing to crack Mr Chisholm's password so that they can then edit their chronicle entries in Compass.

Matthew questions how this alternative method of computation could possibly help them overcome the current limits of computability relating to encryption.

a. Describe what is meant by the phrase 'current limit of computability'. 2 marks

b. Describe **one** way DNA computing could be used to overcome current limits of computation. 3 marks

Unfortunately for Kody and Shiann, they try to use some of Will's DNA and, like Will, it refuses to output anything meaningful. Marcus, who has been watching what is going on carefully just shakes his head in disappointment. Mr Chisholm's password isn't particularly secure. His love for the Melbourne Demons is well documented and from there, realising that his password is simply "GoD33s!" isn't particularly challenging.

After sufficient head shaking, Marcus logs back into Compass and adds some additional negative chronicle entries to Kody and Shiann's portfolios. "That'll keep them busy" he thinks to himself.

Question 15 (5 marks)

Robert and Joshua J are having a discussion with Vincent, Rish and Jessie about the idea of Hilbert's Program when all of a sudden, Finn and Joshua L arrive and are curious about what Hilbert's Program was.

a. In the space below, describe the main goals of Hilbert's Program. 3 marks

Paul and Ranudi arrive and, hearing about what Hilbert's Program is, think that it sounds like a great idea. Kaelen, however, points out that because of Turing's demonstration of the Halting Problem, Hilbert's Program will never be possible.

b. Describe the outcome of Turing's proof of the Halting Problem and its subsequent implications for Hilbert's Program. 3 marks

Vedansh, Nathan, Andrew L, Li Yue, Aden, Nikolai and Karsh all arrive to the discussion and the party really gets started.

Nearby, Wei sees the large gathering and is disappointed that he wasn't invited. After all, he has an excellent understanding of the Universality of Computation and could have answered the groups questions with ease. In retribution for being left out, Wei calls the police and Vedansh, Nathan, Andrew L, Li Yue, Aden, Nikolai, Karsh, Paul, Ranudi, Kaelen, Robert, Joshua J and Joshua L, Vincent, Rish and Jessie are all fined for not adhering to lockdown restrictions.

Somehow Finn avoids getting fined. Noone knows why this is the case but Vedansh theorises that it has something to do with his name being only one vowel short of a Fine.

END OF QUESTION AND ANSWER BOOK