JOHN MONASH SCIENCE SCHOOL

VICTORIAN CERTIFICATE OF EDUCATION
2018

STUDENT NAME:

TEACHER NAME: **BOHL** **HUYT**

ALGORITHMICS (HESS)

Practice Exam 2

(STUDENT DESIGNED VERSION)

2018

Reading Time: 15 minutes
Writing time: 120 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	16	16	80
		Total	100

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 31 pages.
- Answer sheet for multiple-choice questions.

Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice Questions

Instructions for Section A

Answer **all** questions in pencil on the answer provided for multiple-choice questions.
Choose the response that is **correct** or that **best answers** the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \quad \text{where } a > 0, b > 1, c \ge 0, d \ge 0, k > 0$$

and its solution $T(n) = \begin{cases} O(n^c) & \text{if } \log_b a < c \\ O(n^c \log n) & \text{if } \log_b a = c \\ O(n^{\log_b a}) & \text{if } \log_b a > c \end{cases}$

Question 1

Binary Search is an example of:

A. Divide and conquer

B. Decrease and conquer

C. Dynamic Programming

D. Randomised Heuristics

Question 2

Simulated Annealing could be used to attempt to:

A. Determine what move David should make in a game of connect-4.

B. Determine the order of a list

C. Find the n^{th} Fibonacci Number

D. Find the shortest route between n cities for Jeff's Corn Dog Trucks to visit

Question 3

Timsort utilizes merge sort and quicksort in order to minimize both space and time complexity. Hence, Timsort utlisies which of the following advanced algorithm designs?

A. Divide and Conquer

B. Decrease and Conquer

C. All of the above

D. None of the above

Question 4

A pet shop in Clayton receives a shipment of military-grade puppies. Before a puppy is put into the window to be sold its unique name is recorded along with specific information about the puppy: weight, breed, colour, vaccination status, etc.

Which ADT would be the most suited for storing this information?

A. A Dictionary

B. A Queue

C. A Graph

D. An Array

Question 5

An unspecified ADT, X, is initialised with the values {4, 4, 6, 2} the following operations are carried out in the order specified:

X.push(2)

X.pop()

X.peek()

X.push(4)

X.peek()

Which of the following best describes X after these operations have occurred?

A. {4, 4, 4, 6}

B. {4, 4, 6, 2}

C. {4, 4, 4, 6, 2}

D. {4, 4, 6, 2, 2}

Question 6

DNA Computing is sometimes advantageous because it allows for:

A. Massive parrallelism

B. Improved time complexity

C. Improved space complexity

D. Greater ease of implementation

Question 7

According to the Church-Turing Thesis, which of the following statements in **false**?

A. Any problem which can be solved by lambda calculus is computable

B. Any problem which can be solved on a Turing Machine is computable

C. A problem which does not complete in finite time is not computable

D. Any computing machine which can simulate a Turing Machine can solve any computable problem.

Question 8

A Turing Machine

A. Reads and writes onto a tape

B. Reads and manipulates a string of symbols according to a predefined set of rules

C. Given infinite time and space, can solve any computable problems

D. All of the above

Question 9

A neural network is different to traditional algorithms in the way that it:

A. employs highly parallel processing to improve running time

B. does not require explicit design of heuristics to solve the problem

C. improves the time complexity of NP problems

D. continually improves itself as it runs

Question 10

Which proof is used to determine the undecideability of the Halting Problem?

A. Proof by deduction

B. Proof by example

C. Proof by contradiction

D. Proof by induction

Question 11

Worst-Case, Best-Case and Worst-and-Best-Case time complexities are indicated by the symbols:

A. O, Θ, Ω

B. O, Ω, β

C. O, Ω, Θ

D. Θ, O, β

Question 12

Dominic is given a problem and is designing an algorithm to solve the problem for him. He is given an infinite amount of time to find the shortest path from his home city (Melbourne) to Perth. Which algorithm design pattern and algorithm is best suited to solve this problem?

A. Brute Force and Prim's Algorithm

B. Brute Force and the Bellman-Ford Algorithm

C. Greedy and Prim's Algorithm

D. Greedy and the Bellman-Ford Algorithm

Question 13

For extremely large values of n, which one of the following statements in relation to the time complexity of an algorithm is true?

A. An algorithm that runs in $O(n!)$ time is always slower than an algorithm that runs in $O(n^3)$ time.

B. The best case running time of an algorithm that runs in $O(n\log(n))$ time can be faster than the worst case running time of an algorithm that runs in $O(n)$ time.

C. An algorithm that runs in $\Omega(\log(n))$ is always slower than an algorithm that runs in $O(n)$ time.

D. The worst case running time of an algorithm that runs in $O(n!)$ time can be faster than the best case running time of an algorithm that runs in $O(n^2)$ time.

Question 14

What is the worst-case time complexity of an algorithm with recurrence relation $T(n) = 8n + 4T\left(\frac{n}{2}\right)$?

A. $O(n^3)$

B. $O(n\log(n))$

C. $O(n^2)$

D. $O(\log(n))$

Question 15

Consider the pseudocode provided for the following algorithm:

```
Input:    An array A[0…n-1]
Output:   Returns True is all the elements in A are distinct and
          false otherwise.
for i = 0 to i = n-2:
     for j = i + 1 to j = n-1:
          if A[i] = A[j]
               return False
return True
```

Which one of the following correctly identifies the base operation of this algorithm?

A. The inner for loop

B. The comparison between A[*i*] and A[*j*]

C. The return statement

D. The addition of +1

Question 16

An algorithm with worst-case time complexity of $O(n!)$ is most likely to operate on:

A. all permutations of a set

B. all combinations of a set

C. every pair of elements of a set

D. each element of a set

Question 17

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A.

B.

C.

D.

Question 18

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A.

B.

C.

D.

Question 19

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A.

B.

C.

D.

Question 20

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A.

B.

C.

D.

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided

Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \quad \text{where } a > 0, b > 1, c \geq 0, d \geq 0, k > 0$$

and its solution
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Question 1 (2 marks)

The pet shop in Clayton that receives shipments of military-grade puppies has identified that some puppies do not like one another whilst others are best friends. Describe an approach that could be used to identify which puppies can be placed together and which cannot. As part of your answer, describe what ADT you would use to model the problem.

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Question 2 (13 marks)

Riley Purcell likes to wear crazy socks. So much so that he has decided to remove all John Monash Science School sanctioned socks from his drawers, and furthermore, sort his socks into colours of alphabetical order (eg: blue socks would be sorted before red socks and yellow socks would be sorted after blue and red socks. Thomas Reyment realised that Riley was too incompetent to sort his own sock drawer and being the helpful man he is, wrote the following algorithm to help a brother out:

```
Algorithm: sort_Rileys_socks(socks):
     Input:    A one-dimensional array, socks, where each
               element represents a socks colour.
     Output:   A one-dimensional array, socks, with socks sorted
               into colour order.

     for i = 0 to i = number of socks - 1:
          if socks[i] is JMSS Sanctioned:
               remove socks[i] and burn it
          min = i
          for j = 1 to j = number of socks:
               if socks[j] is alphabetically before socks[min]
                    min = j
          swap positions of socks[i] with socks[min]
     return True
```

a. Find the time complexity of the algorithm sort_Rileys_socks. 2 marks

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b. Niamh, curious to how Riley manages his extensive sock collection, asks to see Thomas' sock sorting algorithm. To Thomas' dismay, she takes offence to his use of nested for-loops.
Niamh claims *"These nested for-loops make this algorithm super slow! If you used multiple loops following each other, it would run significantly faster"*

Is Niamh correct? Explain your answer, comparing the complexities of the techniques. 3 marks

c. Diana Lautenbach takes a look at Thomas' algorithm, because she is in Algorithmics and doesn't really have anything better to do. After reading the algorithm, Thomas and Diana proceed to argue over the classification of its time complexity:

Thomas claims *"This problem has P class time complexity."*

Diana states *"I don't really know what's going on but my dad told me the algorithm is NP - Complete as a solution can be verified in polynomial time."*

Who is correct? Explain your answer. 3 marks

d. Even though no one asks for Diana to give her input, as a result of her superiority complex she has developed from her algorithmics class, she proposes that the sock draw can be solved in $O(n\log(n))$ time. Write the name of the sorting algorithm Diana is referring to below. 1 mark

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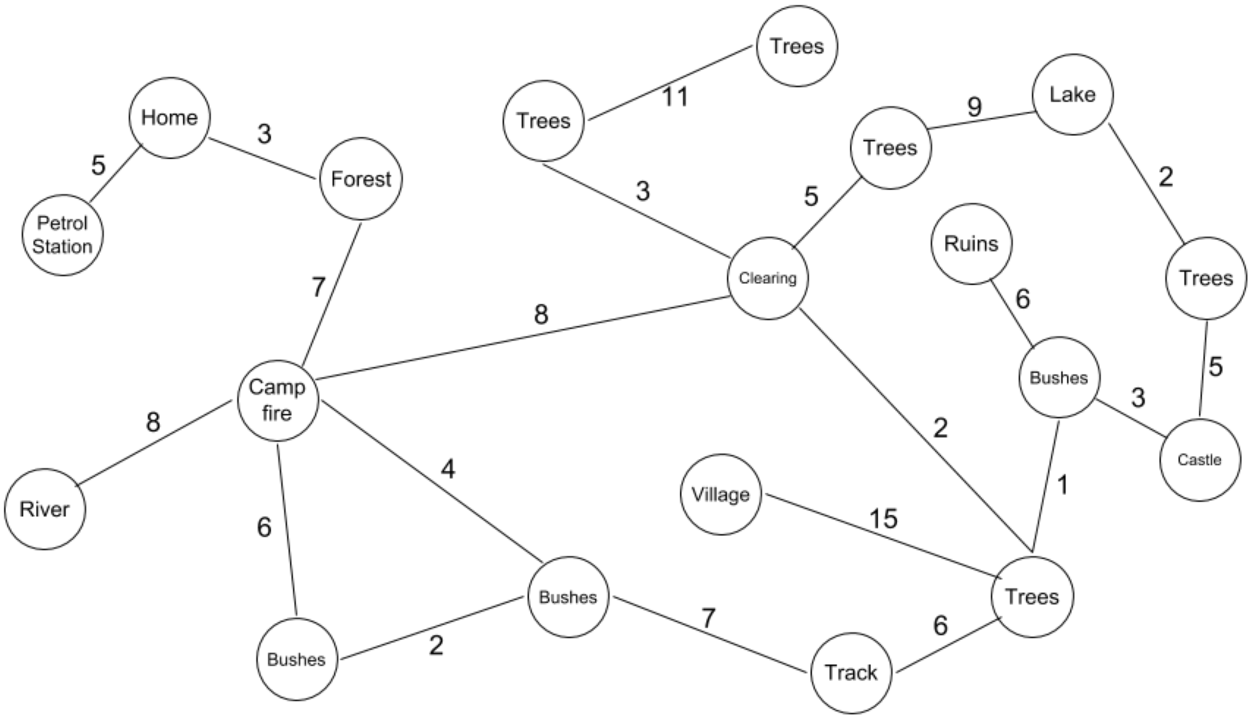
e. Using your answer from part D, write out the pseudocode for the sorting algorithm below. 4 marks

Question 3 (7 marks)

Johnbob the Forest Legend is the most experienced Forest Ranger known to mankind. He's lived next to the forest all his life. No wife, no kids, just him and the forest. He knows the ins and outs of the forest like nobody, not even the forest creatures that call it home. Some people say "that man has the map of the forest in his head!" Some even go as far as saying "he is... THE FOREST..." No one actually knows how he came about; his family and origins all unknown to this day. A legend surfaced; man born from the forest, with only one purpose, to serve and protect. Man or not, he is THE FOREST.

After a busy day's work, Johnbob heads home. Upon entering the household, he realises that Bobjohn, his pet squirrel is not with him. After the anagnorisis, Johnbob figures that his squirrel may be in the deepest part of the forest. He writes down the following map from his memory. Before heading out in search for Bobjohn, Johnbob decides to get wasted to relieve the stress. Intoxicated, Johnbob needs your algorithmic skills to help find his lost pet Bobjohn. He thinks of the map below.



a. Johnbob has enough fuel to travel x km. He first needs to calculate the shortest distance to each node from his home. Which algorithm would he use and why? 2 marks

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b. What is the width of this graph? 1 mark

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c. Should Johnbob refuel at the petrol station before entering the forest and why/why not? 2 marks

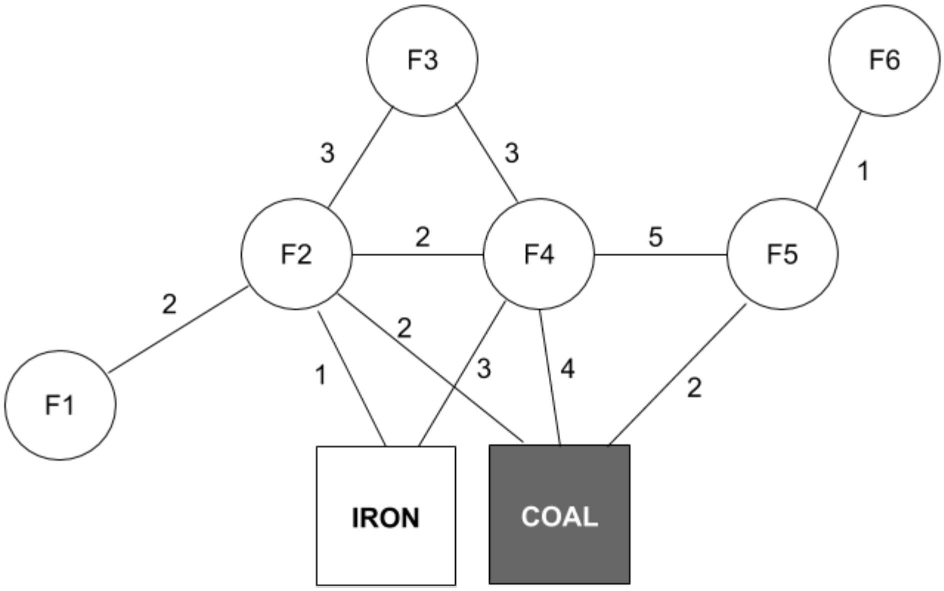
After finding Bobjohn at the deepest part of the forest, Johnbob realises that he forgot to account for enough fuel to get back home. Thus, he sleeps in the forest and recalls the memories of his childhood abandonment.

d. Describe the design pattern that the algorithm in **part a**. uses, stating it's advantages and disadvantages. 2 marks

Question 4 (3 marks)

Ben Sutherland was playing a game of Factorio instead of doing his classwork. Ryan looked at Ben's game and decided that all his ratios were off and that he needed to rewire some of his conveyor belts in order to make a more efficient factory. Each furnace needs to be connected to one coal deposit and one iron deposit in order for Ben to make his favourite item; the iron gear wheel.

The diagram below shows an example of Ben's game.



Describe an algorithm that minimises the amount of conveyor belts required to connect each node to the network of belts and shows the answer on the graph above.

Question 5 (3 marks)

Jeff is hosting a party. Unfortunately, many of his friends are brutal enemies. If he invites them both to the same party, they will try and fight to the death, which Jeff doesn't want to happen. With divine insight, Jeff figures out that if he hosts two separate parties, there would be no conflicts. He hires a secret investigator to obtain a list of guests who are incompatible with each other. Jeff decides that he wants to invite as many people as possible to the first party. Write an algorithm that figures out the maximum possible amount of people that can be invited to the first party, given a list of incompatible pairs.

Question 6 (3 marks)

Consider the following algorithm.

```
Algorithm: Algo1(n)
     if n = 1:
          return -1
     Else:
          If n is odd:
               return (-n + Algo1(n-1)
          Else:
               return (n + Algo1(n-1)
```

a. Convert **Algo1** into a tail recursive algorithm. 2 marks

b. What advantages does the tail recursive algorithm have over the recursive version? 1 mark

Question 7 (5 marks)

Sai wanted to create a program that would allow him to hack into the Eduroam network, but he doesn't know where to start. He desires a program with various utilities that more often than not use the same base utilities.

a. Which approach would be more efficient; a top down approach or a bottom up approach? Justify your response by comparing the two techniques. 3 marks

b. Hugh was angry at the extent we went to create an interesting backstory to our questions and he seeks an algorithm to cut down on the words in the questions. What heuristics can he use in his algorithm? 2 marks

Question 8 (10 marks)

a. Compare the techniques used in the Bellman Ford algorithm and Floyd Warshall algorithm to calculate the distance from one node to every other and all nodes to all other nodes respectively. 3 marks

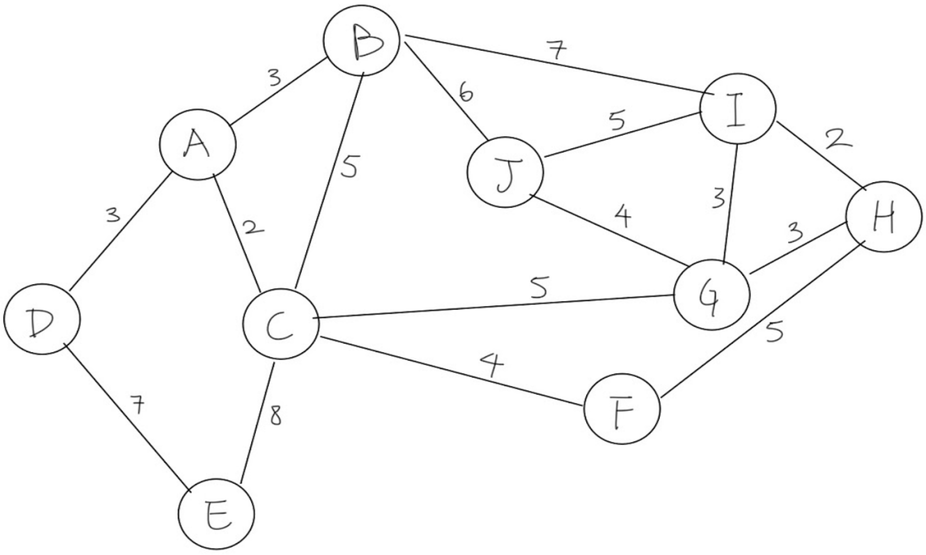
b. Jeff wants to create a path finding algorithm for Euro Truck Simulator 2, but he was told that Bellman Ford and Floyd Warshall are way to slow. He wants to find the guaranteed shortest route from his starting point to another location in Europe. He is also too lazy to learn A*. What ADT is most suited in modelling the cities and roads? What algorithm should he use? 2 marks

c. Jeff decides to compare the performance of Floyd Warshall and Dijkstra's algorithm for the problem above. How can he modify Floyd Warshall's algorithm to not only calculate the shortest path from each node to every other node, but also to store the shortest path? To avoid blowing up his 1GB of RAM, he can only use O(V) memory to store the paths 3 marks

d. How would Jeff be able to detect whether a graph is a tree or not? 2 marks

Question 9 (4 marks)

Consider the graph shown below:



a. What is the function of the Bellman-Ford Algorithm? 1 mark

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b. Starting from node D, after the third iteration (assuming constant update after each loop) of the Bellman-Ford Algorithm identify which of the nodes' distance have been confirmed. Explain why this is the case. 4 marks

A	B	C	D	E	F	G	H	I	J

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Question 10 (2 marks)

As part of a corny joke, Jeff is trying to make his way through a maze of maize. Describe an algorithm design pattern that Jeff could use to solve his problem.

Question 11 (2 marks)

Describe why the Knapsack Problem is considered NP-Hard and provide an NP variant of the problem.

Question 12 (6 marks)

A number, *c*, is considered a cool number if $c + 1$ can be expressed as the product of two different cool numbers. 2 and 3 are the lowest cool numbers despite note displaying this property.
Using a dynamic programming approach, design an algorithm that determines if a positive integer, *n*, is a cool number.

Question 13 (4 marks)

John Searle's Chinese Room Argument examines the nature of artificial intelligence.

a. Present an argument **against** Searle's position. 2 marks

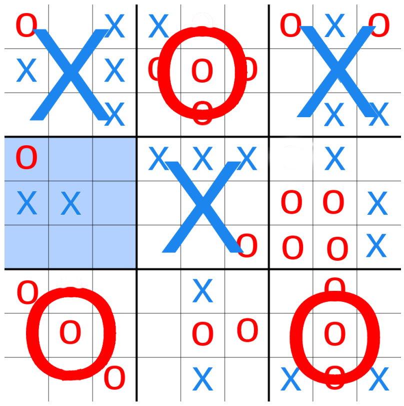
b. Present a **refutation** of the argument presented in **part a.** 2 marks

Question 14 (4 marks)

List and discuss the significance of **two** limitations of Neural Networks.

Question 15 (6 marks)

Mr Bohni and Mr Huynh instead of deciding to teach today; sick of the study design, decide to organise an ultimate tic-tac-toe tournament. After some dodgy decisions, blatant bribes, and falsified forfeits Mr Bohni and Mr Huynh both make it to the final, Mr Bohni is playing as crosses.



Utilising an appropriate algorithm use a tree to determine what move Mr Bohni should make next and what is the best outcome for him, given the above state of play (Mr Bohni has to go in the blue highlighted square)?

Question 16 (6 marks)

Callum wants to sort his collection of Star Trek bootlegs from A-Z. He is thinking of trying either Mergesort or Quicksort.

a. Describe the differences between Mergesort and Quicksort approaches. As part of your answer compare the algorithmic design patterns used by each and their relative time complexities. 3 marks

Callum instead decides to come up with his own, heuristic based algorithm to solve his problem.

b. Describe a possible limitation of using a heuristic based algorithm and a way that this limitation might be compensated for. 3 marks

END OF QUESTION AND ANSWER BOOK