JOHN MONASH SCIENCE SCHOOL

VICTORIAN CERTIFICATE OF EDUCATION

2018

STUDENT NAME:

TEACHER NAME: **BOHL** **HUYT**

ALGORITHMICS (HESS)

Practice Exam

2018

Reading Time: 15 minutes
Writing time: 120 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	12	12	80
		Total	100

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 31 pages.
- Answer sheet for multiple-choice questions.

Instructions

- Write your name in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

This page has been left blank in honour of Tom
(who had to appear on the exam somewhere)

SECTION A – Multiple-choice Questions

Instructions for Section A

Answer **all** questions in pencil on the answer provided for multiple-choice questions.
Choose the response that is **correct** or that **best answers** the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \quad \text{where } a > 0, b > 1, c \ge 0, d \ge 0, k > 0$$

and its solution $$T(n) = \begin{cases} O(n^c) & \text{if } \log_b a < c \\ O(n^c \log n) & \text{if } \log_b a = c \\ O(n^{\log_b a}) & \text{if } \log_b a > c \end{cases}$$

Question 1

One benefit of using Neural Networks as an alternate method of computation is that it allows for:

A. large amounts of parallel processing.

B. the algorithm to be trained to do tasks that are traditionally very hard to write a traditional algorithm to solve.

C. easy interpretation of the relationships between given inputs and the outputs.

D. fast and efficient calculations to be carried out sequentially.

Question 2

For extremely large values of *n*, which of the following statements in relation to the time complexity of the algorithm is correct?

A. The best case time complexity of an algorithm that runs in $O(n^2)$ time can be faster than the worst case time complexity of an algorithm that runs in $O(\log n)$ time.

B. The best case time complexity of an algorithm that runs in $O(n^2)$ time will always be slower than the worst case time complexity of an algorithm that runs in $O(\log n)$ time.

C. The worst case time complexity of an algorithm that runs in $O(n^2)$ time can be faster than the worst case time complexity of an algorithm that runs in $O(\log n)$ time.

D. The worst case time complexity of an algorithm that runs in $O(\log n)$ time will always be slower than the best case time complexity of an algorithm that runs in $O(n^2)$ time.

Question 3

Consider the following function mystery(n, a) that takes two integers, n and a as input.

```
def mystery(n, a)
     if n equals 0
          return a
     return mystery(n - 1, n + a)
```

The above pseudocode is an example of:

A. Standard recursion

B. Tail recursion

C. Iteration

D. Application of the Master Theorem

Question 4

Church and Turing defined the computability of a function by a human and made the assumption that

A. a finite amount of time and space was available to the human

B. a finite amount of time was available to the human

C. a finite amount of space was available to the human

D. the human would make no errors

Question 5

What values of a, b and c in the recurrence relation $T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases}$ would give the running time $T(n)$ as $O(n^2)$

A. $a = 3,\ b = 2,\ c = 4$

B. $a = 1,\ b = 1,\ c = 1$

C. $a = 4,\ b = 2,\ c = 1$

D. $a = 1,\ b = 3,\ c = 3$

Question 6

You wish to find all the nodes within a set distance from another node in a connected network graph. All the edges on the graph are equally weighted.

The most time-efficient algorithm to use from the list below is:

A. Dijkstra's

B. Bellman-Ford

C. Floyd-Warshall

D. Breadth-First search

Question 7

One advantage of a dynamic programming algorithm design is:

A. It trades lower time-complexity for higher space-complexity

B. It trades lower space-complexity for higher time-complexity

C. It applies parallel processing to improve time-complexity

D. It uses biological processes as computational processing

Question 8

Quick sort is an example of which type of algorithm design:

A. Decrease and conquer

B. Greedy

C. Brute force

D. Divide and conquer

Question 9

An algorithm has a recurrence relation given by:

$$T(n) = n^2 + 8T\left(\frac{n}{2}\right).$$

Which of the following represents the Big-O time complexity of this algorithm?

A. $O(n^2 \log n)$

B. $O(n^2)$

C. $O(n^3)$

D. $O(n^{\log n})$

Question 10

Which ADT is most time-efficient for the random removal of elements in a collection?

A. Dictionary

B. Queue

C. Stack

D. Tree

Question 11

Which of the following is **not** a true statement relating to John Searle's Chinese Room Argument?

A. The Chinese Room Argument proposes that computers are only ever capable of achieving Weak AI.

B. The Chinese Room Argument supports the idea of computers being incapable of achieving Strong AI.

C. The Chinese Room Argument is an argument that computers are not capable of ever understanding what they are doing.

D. The Chinese Room Argument is an attempt to define what artificial intelligence is and is not.

Question 12

Which types of prediction problems are Neural Networks most optimized for:

A. Clustering

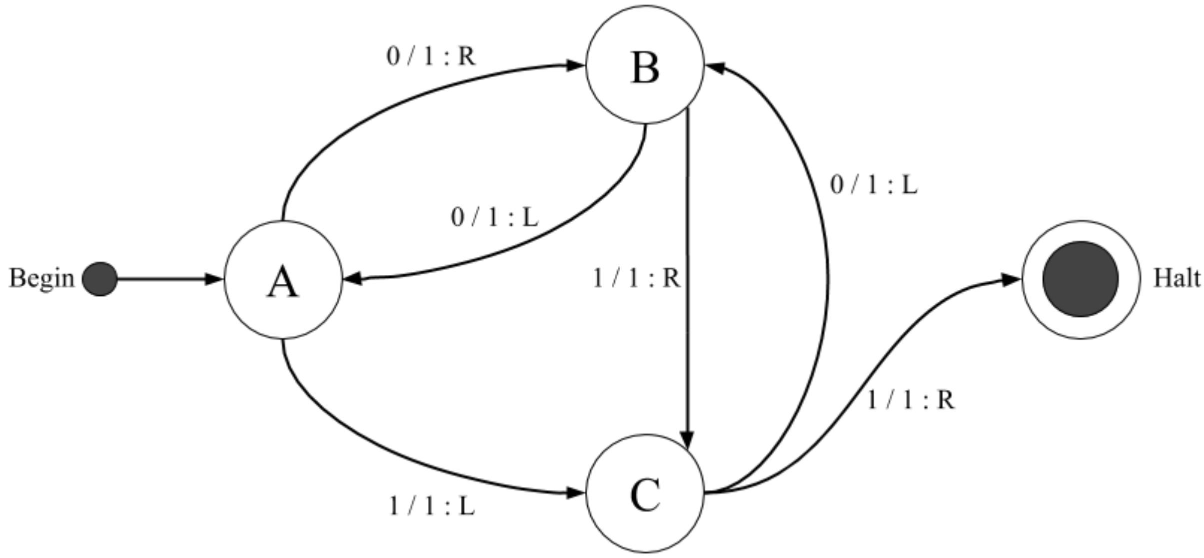
B. Back-propagation

C. Machine learning

D. Classification

Use the following information to answer Questions 13 and 14

A Turing Machine is configured with the instructions represented in the state diagram below.



Each edge is labelled *i* / *j* : *k*, where:

- *i* is the input
- *j* is the output
- *k* is the direction the head moves (L = left, R = right) after the output.

The machine starts in state A.

The machine is given the following tape. For this machine, the tape remains stationary while the head moves. The arrow shows the starting point of the head.

0	0	0	0	0	0	0	0	0	0
				↑					

Question 13

The Turing Machine is run for 4 steps. In what state will the Turing Machine be in at this time?

A. State A

B. State B

C. State C

D. It will be in its Halt State.

Question 14

Which one of the following best represents the tapes appearance and the position of the head when the Turing Machine halts?

A.

0	0	1	1	1	1	0	0	0	0
				↑					

B.

0	0	1	1	1	0	0	0	0	0
					↑				

C.

0	0	0	0	1	1	1	0	0	0
							↑		

D.

0	0	0	0	1	1	0	0	0	0
						↑			

Question 15

Consider the functions:

$$f(n) = 2n^3 + n$$
$$g(n) = n \log n + 2n^2$$
$$h(n) = \frac{f(n)}{g(n)}$$
$$j(n) = f(n) + g(n)$$

What is the best Big-O time complexity description of the asymptotic growth rate of $h(n)$ and $j(n)$ respectively?

A. $O\left(\frac{n^2}{\log n}\right)$, $O(n^3)$

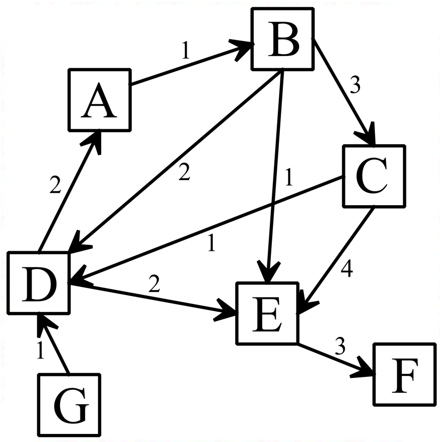
B. $O(1)$, $O(n)$

C. $O\left(\frac{1}{n^2}\right)$, $O(n^2)$

D. $O(n)$, $O(n^3)$

Question 16

Consider the network graph shown below:



The width of the graph is:

A. 7

B. 8

C. 6

D. 4

Question 17

The following is the pseudocode for a Bellman-Ford algorithm that contains an error:

```
for each vertex v in vertices:
    distance[v] = ∞
    predecessor[v] = null
Distance[source] = 0
for i from 1 to size(vertices)-1:
    for each edge (u, v) with weight w:
        if distance[u] + w > distance[v]:
            distance[v] = distance[u] + w
            predecessor[v] = u
```

Which of the following statements correctly identifies the error in this code?

A. Line 2: inf should be 0

B. Line 4: 0 should be ∞

C. Line 7: > should be <

D. Line 8: + w should be -w

Question 18

What is a difference between the List and Array ADTs?

A. Nothing, they are the same, just implemented differently in programming languages

B. Arrays store their elements contiguously in memory while lists don't

C. Lists can't store elements of different types while Arrays can

D. Stacks and Queues can be implemented using lists. They cannot be implemented using Arrays.

Question 19

Which of the following descriptions provides the most accurate description of the Halting Problem?

A. The Halting Problem is the problem of determining whether an algorithm will halt or run forever.

B. The Halting Problem is the problem of identifying which inputs will result in an algorithm being undecidable or not.

C. The Halting Problem is the problem of determining whether an algorithm, with specific input, will halt or run forever.

D. The Halting Problem is a proof that a general algorithm to determine whether all possible program-input pairs will or will not halt, cannot exist.

Question 20

Sai makes the following statements about the Bellman-Ford Algorithm on a graph with n nodes:

I Its running time is in the P class

II It is an example of an algorithm that makes use of dynamic programming

III It has a worst case time complexity of $O(n^2)$

IV It has a best case time complexity of $\Theta(\log n)$

Which group of statements is correct?

A. I and III

B. I, II and III

C. I, III, IV

D. I and IV

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided

Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \quad \text{where } a > 0, b > 1, c \geq 0, d \geq 0, k > 0$$

and its solution
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Question 1 (3 marks)

Kade has been given the role of Deputy Operational Performance Enforcer (DOPE) at the local Christmas Tree farm. In this role, he is required to place 'watchers' in specific paddocks around the farm to ensure that no illegal activities occur in each of these paddocks. When a watcher is in a given paddock, they can supervise that paddock as well as all immediately adjacent paddocks.

Outline an algorithm that Kade could use to determine the placement of watchers in paddocks around the farm in a reasonable amount of time. As part of your answer, identify an appropriate abstract data type (ADT) that could be used, referring to its specifications.

Question 2 (6 marks)

Sai has recently given up on world domination due to an overwhelming feeling of 'meh'. Instead, he has focussed his attention on his colony of fire ants (they were an important part of his original plan to take over the world but that's not really relevant anymore).

The fire ants have created a colony of rooms that are connected by tunnels. Sai wishes to make his ants into a species of super ants through the application of gamma rays but has noticed that the fire ants have a tendency to burst into flames when irradiated. As a result, he has installed water sprinklers in each of the rooms in the colony as a way of trying to minimise the spread of any fires that might start.

When one of the fire ants bursts into flames, the following algorithm is used to determine which rooms in the colony the sprinklers should be turned on in so that the flames can's spread further.

```
Algorithm Hydrate(L)
input:   L a list of all of the rooms that are connected by a
         tunnel to the room in which the fire started in order of
         distance from the starting room.

While L is not empty
         Engage Sprinklers in the first element of L
         Remove the last element in L
```

a. The proposed algorithm has errors.

How many rooms in the fire ant colony will have the sprinklers turned on when a fire ant bursts into flames in a room which is connected to 6 adjacent rooms? 1 mark

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b. Write a corrected version of the algorithm that will allow the sprinklers to be turned on in all of the adjacent rooms to the room where the fire ant burst into flames. 2 marks

c. Sai notices that when the sprinklers turn on, the water begins to spread through the ant colony, flooding all of the adjacent rooms and extinguishing the lives of his precious fire ants.

What ADT would be most appropriate for modelling the rooms of the ant colony? Assuming that the water from the sprinklers spreads to all connected rooms to the one where the sprinkler was turned on, what type of algorithm could be used to with your selected ADT to model the flow of the water through the ant colony? 3 marks

Sai was recently reported to animal cruelty by Rohan and sent to prison for a life sentence as the jury at his trial had a soft spot for fire ants.

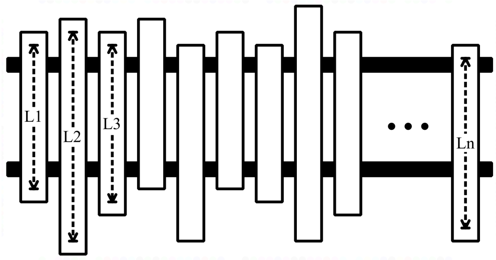
Rohan has since adopted those fire ants that survived the 'incident' (as it is now referred to) and is utilising the fire ants in his own evil plans to take over the world.

Little does he know that one of the fire ants in his collection is actually Marcus in disguise… If you are having trouble understanding how this is possible, you should probably move on to the next question and stop wasting time on random story arcs that have no relevance to any marks.

Question 3 (12 marks)

Sean and Parker have been given a work management by Mr Chisholm for claiming that the Melbourne Demons weren't the best team in the AFL (an obvious fallacy). As part of their punishment, they have been told to organise the painting of a new picket fence (in red and blue of course).

There are 5 painters available and each painter takes 1 unit of time to paint 1 unit length of fence. The fence is made up of vertical pickets of length {L1, L2, L3,… Ln}.



Sean and Parker wish to find the minimum time required to get the fence painted given that a picket can only have one painter and any given painter will only paint adjacent pickets (for example, an individual painter might paint pickets 2, 3 and 4 but they won't paint pickets 2, 4 and 6).

a. Write a brute-force algorithm that will return the minimum time required to paint the fence. 4 marks

SECTION B - continued

b. Riley, who was also involved in spreading blatant lies about the Melbourne Demons but who didn't get caught by Mr Chisholm, was feeling guilty and so wrote the following two algorithms to try and help Sean and Parker with their problem.

```
SumsAlgo(A[ ], start, end)
     Input A[ ]: An array of the lengths of each picket in
     order.
     Input start: an integer value indicating the index in A
     from which to begin.
     Input end: an integer value indicating the index value in A
     at which to end.

     Total = 0
     for i = start to i = end
          Total = Total + A[i]
     return Total

RileysAlgorithm(A[ ], n, k]
     Input A[ ]: An array of the lengths of each picket in
     order.
     Input n: The total number of pickets in the fence.
     Input k: The number of painters that are available to paint
     the picket fence.

     Let Split = ⌊ n / k ⌋
     Best = 0
     for i = 0 to i = k - 1
          SumsAlgo(A[ ], i*(Split + 1), (i+1)*(Split+i))
          if Total > Best
               Best = Total
     return Best
```

i. Which algorithmic design patterns has Riley made use of in the design of his algorithms? 2 marks

ii. Write a recurrence relation that reflects the running time of RileysAlgorithm as it is written. 2 marks

c. Compare a brute force approach like the one used in **part a** with the approach used by Riley in **part b**.
Which approach should Sean and Patrick utilise in solving this problem. Justify your answer with appropriate reasoning. 4 marks

This page is a joke.

Xavier, Mark and Scott all walked into a bar…

… you'd have thought that at least one of them would have seen it and ducked.

Question 4 (9 marks)

Aaron is wanting to compute the shortest path for his video game character to journey from the town tavern to the Caves of Doom. He knows that locations in his game are represented as nodes in a graph and their distances are represented as weights on the connected edges. Aaron wishes to use Dijkstra's algorithm to find the shortest route for his game character to traverse.

a. By applying a Proof by Contradiction, prove that Dijkstra's algorithm will always correctly determine the shortest path between two nodes on a graph that does not contain any negative edge weights. 4 marks

b. Unfortunately, the character never seems to budge from his initial position, and Aaron suspects that there is a negative cycle somewhere in his graph configuration.

He remembers how the Floyd-Warshall algorithm can be adapted to detect negative cycles in graphs and decides to give this a try.

In pseudocode, write a version of the Floyd-Warshall algorithm that Aaron could use for his problem. 4 marks

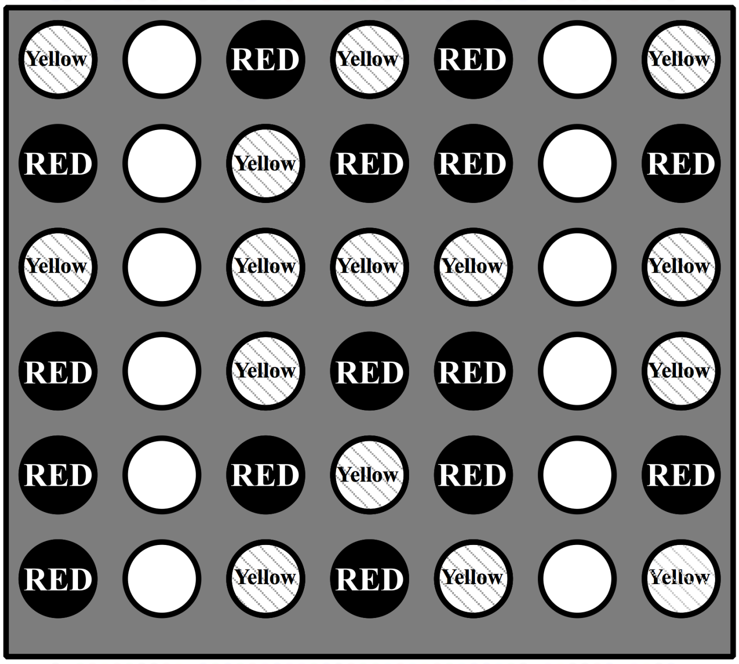
c. Explain how Aaron can adapt the algorithm to detect whether or not there are negative cycles in a given graph. 1 mark

Question 5 (8 marks)

The game of Connect-4 is a ***solved*** game, which means that an outcome can be predicted from any state of the game with the assumption that both players play perfectly.

Below is the current state of the game of Connect-4 between Vlad and David. It is currently Vlad's turn and he is playing as RED*.

*In case you are unsure, as Vlad is RED, it means that David is playing as Yellow.



a. Explain how the Minimax algorithm would go about determining what move Vlad should make as his next move. As part of your answer make sure that you reference any assumptions that the Minimax algorithm would make. 3 marks

b. Draw the Minimax state-space graph the illustrates all of the possible moves from this current state until an outcome is obtained.

Indicate on your state-space graph what move Vlad should make in order to maximise his chance of beating David. 3 marks

c. How would the feasibility of using Minimax to determine the next move change if Vlad were to try and use it from the very beginning of the game? 2 marks

Question 6 (3 marks)

Describe the Chinese Room Argument as defined by John Searle. In your response refer to the ideas of strong AI and weak AI and how the Chinese Room Argument supports or does not support these ideas.

Question 7 (6 marks)

The Church-Turing Thesis and Cobham's Thesis both attempt to define the nature of computability.

a. Outline how the Church-Turing Thesis attempts to define computability and how it was influenced by Hilbert's 1927 Program. 3 marks

b. Outline how Cobham's Thesis attempts to define computability. As part of your answer, discuss some of the limitations of Cobham's Thesis in defining computability in this manner. 3 marks

Question 8 (10 marks)

Daniel has decided to take on Google and write his very own search engine which will return the best results for how to write English essays.

One of the first steps he has identified to write is a web 'crawler' which is a program that will, given a starting web page, follow all of the links from that page and save them on a separate disk.

a. What ADTs would you recommend Daniel utilise when writing this web crawler. 2 marks

As part of his research into designing his very own search engine, Daniel takes a look at the PageRank algorithm and how it works. Thanks to a rather mysterious online acquaintance that Daniel met (who goes by the pseudonym Mr Treyment), Daniel comes across the following equation for PageRank that would act upon a series of 5 webpages, *A*, *B*, *C*, *D* and *E*.

$$\mathrm{PR}(A) = \frac{(1-d)}{N} + d\left(\frac{\mathrm{PR}(B)}{\mathrm{L}(B)} + \frac{\mathrm{PR}(C)}{\mathrm{L}(C)} + \frac{\mathrm{PR}(D)}{\mathrm{L}(D)} + \frac{\mathrm{PR}(E)}{\mathrm{L}(E)}\right)$$

b. **i.** What does the $\mathrm{PR}(A)$ represent? 1 mark

ii. What does the d term represent? 2 marks

iii. What does the $\frac{(1-d)}{N}$ term represent? 1 mark

iv. What does the $L(B)$ term represent? 1 mark

v. What does the $d\left(\frac{\mathrm{PR}(B)}{\mathrm{L}(B)}+\frac{\mathrm{PR}(C)}{\mathrm{L}(C)}+\frac{\mathrm{PR}(D)}{\mathrm{L}(D)}+\frac{\mathrm{PR}(E)}{\mathrm{L}(E)}\right)$ term represent? 1 mark

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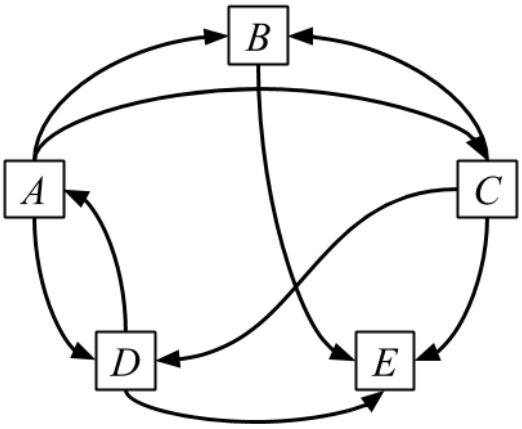
The series of webpages from **part b** can be represented by the following network graph:



c. The webpage *E* has no outgoing links. Describe how PageRank would deal with the values attributed to webpage *E*. 2 marks

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Daniel is so confused he returns to the online forum where he met Mr Treyment and starts up a conversation. They get to talking about such things as life, the Universe and everything. Things are going really well and both Daniel and Mr Treyment seem to be becoming fast friends.

A few days later, Mr Treyment reveals to Daniel that he in fact the prince of a remote country that has recently gone through a rather violent political shake up. As a result ‘Prince’ Treyment is being forced to flee the country. He has a large amount of money that he needs to store in an off-shore account though so that the new political movement in his country doesn’t take it from him. He asks for Daniel’s bank account details that he might deposit a large sum of money in Daniel’s account for safe keeping. As a reward for doing so, Daniel would get to keep half of the money that is deposited.

Daniel, being very excited about his good fortune, tells his friend Chris all about it. Chris quietly excuses himself from his conversation with Daniel and proceeds to go and talk to another Algorithmics student Thomas Reyment. Later, Chris returns and is very encouraging of Daniel’s great fortune and offers to help him in any way that he can… A few years later Daniel is in financial ruin whilst Chris and Tom seem to have been very lucky and are driving around in very fancy (and expensive) cars.

Question 9 (4 marks)

Traveen and Lleyton are designing a program that uses a variety of different algorithms of different time complexities. Lleyton wishes to run the algorithms sequentially, one after the other, whereas Traveen wishes to 'nest' the algorithms with each algorithm calling another as it runs.

Discuss how the variation in complexity can affect the overall running time of the program if they are nested (Traveen's idea) compared with run sequentially (Lleyton's idea). In your response, use examples of time complexity.

Question 10 (8 marks)

Emerging technologies may help to overcome some of the limits of current computing systems.

a. Describe **one** limit that exists in current models of computing. 2 marks

b. Describe **one** way a neural network could be used to overcome current limits of computation. 3 marks

c. Describe **one** way DNA computing could be used to overcome current limits of computation. 3 marks

Question 11 (6 marks)

Ryan and Raj have been asked by the JMSS IT department to help lay out the network cabling for a new supercomputing lab! There will be 7 new PCs (creatively named **a**, **b**, **c**, **d**, **e**, **f**, **g**) in the lab in addition to the supercomputer.

Below is the adjacency matrix that describes the distances, in meters, between the location of each of the computers in the supercomputing lab.

	a	**b**	**c**	**d**	**e**	**f**	**g**
a		4	8				
b	4		9	8	10		
c	8	9		2		1	
d		8	2		7	9	
e		10		7		5	6
f			1	9	5		2
g					6	2	

a. Draw a graphical representation of the connections between the computers in the supercomputing lab. As part of your answer, include a legend explaining what each aspect of the graph represents. 3 marks

b. Ryan and Raj wish to determine what the minimum amount of cabling would be in order to ensure that each of the seven computers, **a**, **b**, **c**, **d**, **e**, **f**, **g**, are connected to the same network.

What algorithm would you recommend for Ryan and Raj use? As part of your answer, provide a reason for why this algorithm would be **most** appropriate. 2 marks

c. What is the minimum amount of cabling required to ensure that each of the seven computers, **a**, **b**, **c**, **d**, **e**, **f**, **g**, are connected to the same network? 1 mark

This page has been left blank in order to give you a moment in which to pause and breathe before attempting the final question.

Of course, if you are running out of time, please stop reading this and get on with it!

Question 12 (5 marks)

Josh, while unwell in hospital was thinking up ways to torment those who were responsible for putting him there. Patrick was at the top of his list.

The problem that Josh came up with was based off the following grid.

3	6	8
9	1	4
7	5	2

Patrick was asked to find the maximum length path, starting from any cell in the grid, such that all of the cells along the path are in increasing order with a difference of 1.

a. Describe an algorithm design pattern that Patrick could use to solve Josh's problem. As part of your answer, make sure that you discuss the features of this problem that make this algorithm design pattern appropriate. 3 marks

b. Josh considered using a larger size grid to torment Patrick but was convinced by Matt that the 3x3 grid would be difficult enough for Patrick. Describe how the difficulty of the problem would be affected by increasing the size of the dimensions of the grid. 2 marks

END OF QUESTION AND ANSWER BOOK