

## QCE Specialist Mathematics Units 3&4

### Paper 2 – Technology-active

#### SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
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7.	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
8.	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
9.	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
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**QUESTION 1 C**

C is correct.

$$\begin{aligned}\varphi &= \tan^{-1} \frac{c}{\sqrt{a^2 + b^2}} \\ &= \tan^{-1} \frac{7}{\sqrt{2^2 + (-3)^2}} \\ &= \tan^{-1} \frac{7}{\sqrt{13}} \\ &= 1.095^\circ\end{aligned}$$

A is incorrect. This option may be reached by using the modulus of the vector,  $\sqrt{62}$ , rather than  $\sqrt{13}$ .

B is incorrect. This option may be reached by calculating the azimuth,  $\theta$ .

D is incorrect. This option may be reached by finding the obtuse angle variation for the altitude.

**QUESTION 2 B**

B is correct.

$$\begin{aligned}\int \frac{7x+3}{(x+3)(2x-3)} dx &\equiv \frac{A}{x+3} + \frac{B}{2x-3} \\ 7x+3 &\equiv A(2x-3) + B(x+3)\end{aligned}$$

When  $x = 1.5$ :

$$7x + 3 = B(x + 3)$$

$$13.5 = 4.5 \times B$$

$$B = 3$$

When  $x = -3$ :

$$7x + 3 = A(2x - 3)$$

$$-18 = -9A$$

$$A = 2$$

$$\text{Therefore, } \int \frac{7x+3}{(x+3)(2x-3)} dx = \int \frac{2}{x+3} + \frac{3}{2x-3} dx$$

A is incorrect. This option may be reached by substituting  $A = -3$  and  $B = 1.5$  into the initial equation.

C is incorrect. This option may be reached by using  $x = -3$  to find  $B$ .

D is incorrect. This option may be reached by making an error when calculating  $A$  using  $-3$ .

**QUESTION 3 A**

A is correct. When  $x = 1$ :

$$y + (4 \times 1)y - 1^2 = 7$$

$$y = \frac{8}{5}$$

$$\frac{d}{dx}(y + 4xy - x^2) = \frac{d}{dx}(7)$$

$$\frac{dy}{dx} + 4y + 4x - 2x = 0$$

$$\frac{dy}{dx}(1 + 4x) = 2x - 4y$$

$$\frac{dy}{dx} = \frac{2x - 4y}{1 + 4x}$$

Substituting  $x = 1$  and  $y = \frac{8}{5}$  gives:

$$\begin{aligned} \frac{dy}{dx} &= \frac{2(1) - 4\left(\frac{8}{5}\right)}{1 + 4(1)} \\ &= -\frac{22}{25} \\ &= -0.88 \end{aligned}$$

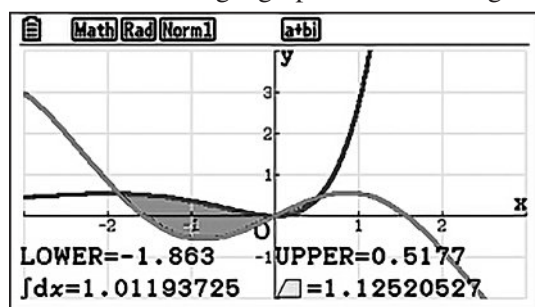
B is incorrect. This option may be reached by finding and substituting  $y = \frac{6}{5}$ .

C is incorrect. This option may be reached by substituting  $x = 1$  and  $y = 1$ .

D is incorrect. This option may be reached by leaving out the term  $4x \frac{dy}{dx}$  in the product rule.

**QUESTION 4 C**

C is correct. Using a graphics calculator gives:



A is incorrect. This option may be reached by calculating the area of the shaded region above the  $x$ -axis.

B is incorrect. This option may be reached by calculating the area of the shaded region below the  $x$ -axis.

D is incorrect. This option may be reached by not treating both parts of the shaded region as positive.

**QUESTION 5 D**

**D** is correct.

$$\frac{dy}{dx} = 6x^2y$$

$$\int \frac{1}{y} dy = \int 6x^2 dx$$

$$\ln y = 2x^3 + c$$

$$y = e^{2x^3 + c}$$

$$= Ae^{2x^3}$$

Thus, any expression in the form  $y = e^{2x^3 + c}$  or  $y = Ae^{2x^3}$ , with any given  $A$  or  $c$ , is a possible solution to this equation.

**A** is incorrect. This option may be reached by differentiating the  $x$  term.

**B** is incorrect. This option may be reached by incorrectly differentiating the power and using the constant term.

**C** is incorrect. This option may be reached by incorrectly applying the constant after transforming from a logarithm to an exponential.

**QUESTION 6 A**

**A** is correct. The system is inconsistent when the determinant is zero. Therefore:

$$\det \begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ k & 0 & 1 \end{bmatrix} = 0$$

$$\text{When } k = -3, \det \begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ -3 & 0 & 1 \end{bmatrix} = 0.$$

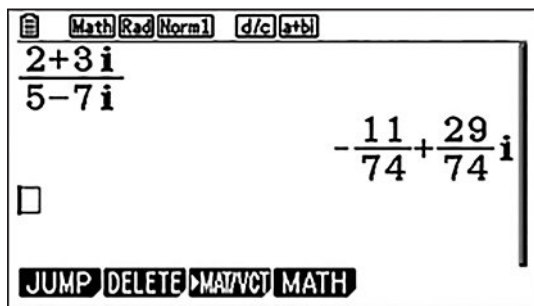
**B** is incorrect. This option may be reached by evaluating  $k$  as 0.

$$\text{C is incorrect. This option may be reached using } \det \begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ 0 & k & 1 \end{bmatrix} = 0.$$

$$\text{D is incorrect. This option may be reached using } \det \begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ k & 1 & 0 \end{bmatrix} = 0.$$

**QUESTION 7 C**

C is correct. Using a graphics calculator gives:



A is incorrect. This option may be reached by inputting  $5 + 7i$  as the denominator.

B is incorrect. This option may be reached by incorrectly using a complex conjugate.

D is incorrect. This option may be reached by making a calculation error using the fraction functionality in the calculator.

**QUESTION 8 C**

C is correct. Using  $\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right) = \frac{a}{a^2 + x^2}$  gives:

$$\begin{aligned}\int \frac{2}{9+x^2} dx &= \frac{2}{3} \int \frac{3}{9+x^2} dx \\ &= \frac{2}{3} \tan^{-1}\left(\frac{x}{3}\right) + c\end{aligned}$$

A is incorrect. This option may be reached by using an incorrect rule from the fractional format.

B is incorrect. This option may be reached by making an error in the calculation or not considering the constant multiple.

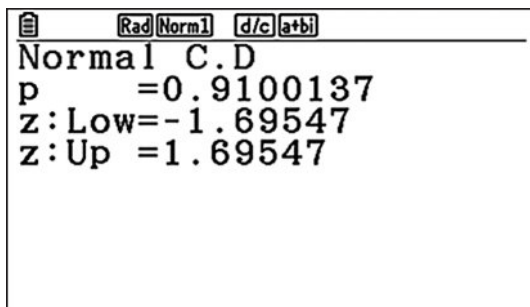
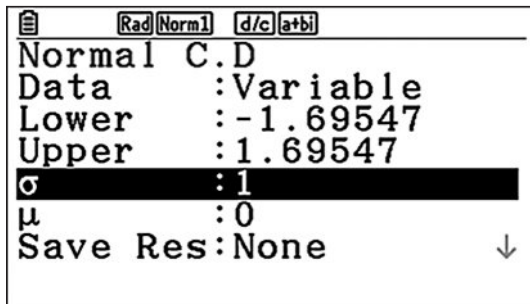
D is incorrect. This option may be reached by confusing the logarithmic and trigonometric inverse laws.

**QUESTION 9 B**

**B** is correct. Finding the quantile gives:

$$\begin{aligned} \text{margin of error} &= z_x \times \frac{s}{\sqrt{n}} \\ \frac{32.2183 - 31.5816}{2} &= z_x \times \frac{2.3}{\sqrt{150}} \\ 0.31835 &= z_x \times \frac{2.3}{\sqrt{150}} \\ z_x &= 1.6954 \dots \end{aligned}$$

Using a graphics calculator gives:



$$\begin{aligned} x &= 0.91 \\ &= 91\% \end{aligned}$$

**A** is incorrect. This option may be reached by incorrectly rounding the solution.

**C** is incorrect. This option may be reached through translation errors.

**D** is incorrect. This option may be reached by using a left tail, rather than a centre approach.

**QUESTION 10 D**

**D** is correct. As  $i = \text{cis}(90^\circ)$ , using De Moivre's theorem shows that multiplying by  $i$  will increase the angle by  $90^\circ$ . Thus, it will cause  $z$  to be rotated counter-clockwise by  $90^\circ$ .

**A** and **B** are incorrect. Multiplying by  $i$  adjusts the argument only.

**C** is incorrect. Multiplying by  $i$  adds  $90^\circ$  to the argument, which moves it counter-clockwise.

**SECTION 2****QUESTION 11 (5 marks)**

- a) The standard periodic function for simple harmonic motion is  $y = A\sin(\omega t + a)$ .

The graph has an amplitude of 2 m; therefore,  $A = 2$  m.

The graph shows three complete cycles over four seconds. Therefore:

$$T = \frac{2\pi}{\omega}$$

$$\frac{4}{3} = \frac{2\pi}{\omega}$$

$$\omega = \frac{3\pi}{2}$$

[2 marks]

1 mark for determining the value of  $A$ .

1 mark for determining the value of  $\omega$ . Note: Accept equivalent values; for example,  $1.5\pi$ .

- b)  $v^2 = \omega^2(A^2 - x^2)$

$$= \left(\frac{3\pi}{2}\right)^2(2^2 - 1^2)$$

$$v = \frac{3\pi\sqrt{3}}{2}$$

$$= 8.1621 \text{ m s}^{-1}$$

[1 mark]

1 mark for determining the value of  $v$ .

Note: Consequential on answer to **Question 11a**).

- c)  $a = -A\omega^2 \sin(\omega t + a)$

$$= -2 \times \left(\frac{3\pi}{2}\right)^2 \sin\left(\frac{3\pi}{2}t\right)$$

$$= -\frac{9\pi^2}{2} \sin\left(\frac{3\pi}{2}t\right)$$

Given that maximum acceleration occurs when  $\sin\left(\frac{3\pi}{2}t\right) = -1$ :

$$\text{maximum acceleration} = -\frac{9\pi^2}{2} \times -1$$

$$= \frac{9\pi^2}{2}$$

$$= 44.41 \text{ m s}^{-2}$$

[2 marks]

1 mark for determining the model for acceleration and recognising the value that maximises the model. Note: The model for acceleration may be implied by subsequent working.

1 mark for providing the correct solution.

**QUESTION 12 (7 marks)**

a) 
$$\bar{x} = \frac{84.6 + 89.2}{2}$$

$$= 86.9$$

[1 mark]

1 mark for determining the sample mean.

b) 
$$z_{.95} \frac{\sigma}{\sqrt{n}} = \frac{89.2 - 84.6}{2}$$

$$1.96 \times \frac{\sigma}{\sqrt{60}} = 2.3$$

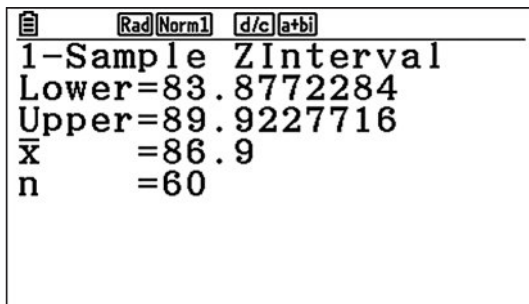
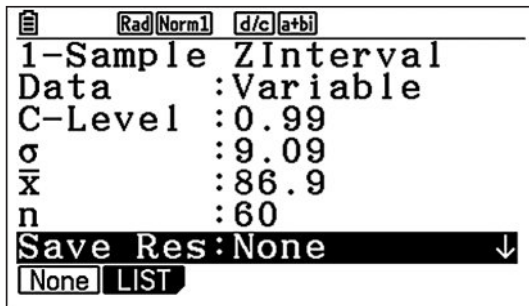
$$\sigma = 9.09$$

[2 marks]

1 mark for using an appropriate formula involving the margin of error.

1 mark for providing the correct solution.

- c) Using a graphics calculator to find the 99% confidence interval, where  $\bar{x} = 86.9$  and  $\sigma = 9.09$  gives:



$CI_{99} = (83.88, 89.92)$

[1 mark]

1 mark for stating the approximate confidence interval. Note: Accept any correct rounding.



d) For the 95% confidence interval of the second sample:

$$\begin{aligned}\text{margin of error} &= 0.7 \times 2.3 \\ &= 1.61\end{aligned}$$

$$n = 60 + x$$

$$\sigma = 9.09$$

Therefore:

$$\text{margin of error}_{95} = z_{95} \frac{\sigma}{\sqrt{n}}$$

$$1.61 = 1.96 \frac{9.09}{\sqrt{60+x}}$$

$$\frac{9.09}{\sqrt{60+x}} = \frac{1.61}{1.96}$$

$$9.09 = 0.8214 \times \sqrt{60+x}$$

$$60+x = \left( \frac{9.09}{0.8214} \right)^2$$

$$x = 62.466 \dots$$

$$= 62$$

[3 marks]

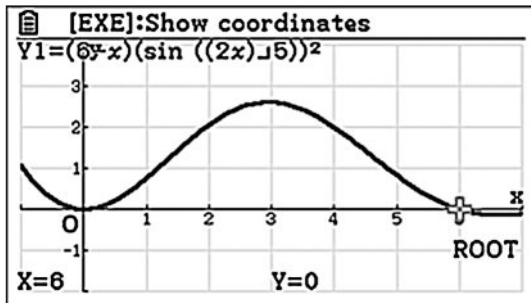
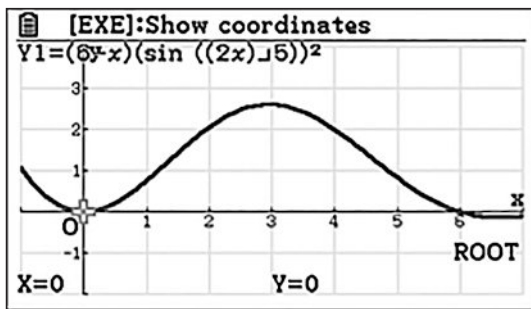
*1 mark for substituting into the margin of error formula.*

*1 mark for calculating the unrounded value of  $x$ . Note: This mark may be implied by subsequent working.*

*1 mark for interpreting that  $x$  must be a whole number and appropriately rounding the value of  $x$ .*

**QUESTION 13 (5 marks)**

- a) Using a graphics calculator shows  $x$ -intercepts at  $x = 0$  and  $x = 6$ , and width of a strip,  $w = 1$ .



Therefore, letting  $w$  be 1 and using the table function in the graphics calculator gives:

[Math] [Rad] [Norm1] [d/c] [a+bl]  

X	Y1
0	0
1	0.7582
2	2.0583
3	2.606

 0  
 FORMULA DELETE ROW EDIT GPH-COIN GPH-PLT

[Math] [Rad] [Norm1] [d/c] [a+bl]  

X	Y1
3	2.606
4	1.9982
5	0.8268
6	0

 6  
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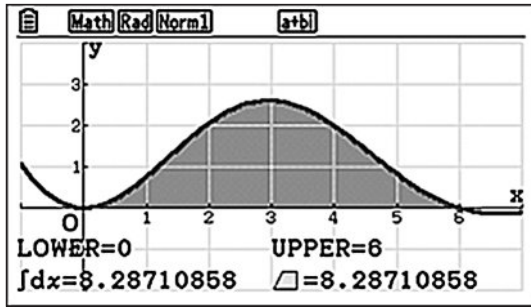
Using Simpson's rule gives:

$$\begin{aligned}
 A &= \frac{w}{3} (h_0 + h_6 + 4(h_1 + h_3 + h_5) + 2(h_2 + h_4)) \\
 &= \frac{1}{3} (0 + 0 + 4(0.7582 + 2.606 + 0.8268) + 2(2.0583 + 1.9982)) \\
 &= 8.2923 \dots \text{ units}^2
 \end{aligned}$$

[4 marks]

- 1 mark for finding the start and end points of the shaded area.
- 1 mark for providing the correct values for all coordinates within the domain.
- 1 mark for substituting into the Simpson's rule formula.
- 1 mark for providing the correct solution.

- b) Using a graphics calculator to find the exact area gives:



$$A = 8.2871\dots \text{ units}^2$$

The area found using Simpson's rule is out by  $0.052\dots \text{ units}^2$ , or  $0.063\%$ ; therefore, the approximation is very accurate.

[1 mark]

1 mark for providing the exact area under the curve and drawing a conclusion regarding reasonableness.

Note: Consequential on answer to **Question 13a**).

**QUESTION 14 (6 marks)**

a)

$$D_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

[1 mark]

1 mark for stating  $D_1$ .

b)

$$D_1 \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

First place (tied): team B and team C

Second place: team A

Third place (tied): team D and team E

[1 mark]

1 mark for listing the ranking, including any ties that occurred.

Note: Consequential on answer to **Question 14a**).

$$\begin{aligned}
 \text{c)} \quad \left(D_1 + \frac{1}{2}D_2\right) \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} &= \left(D_1 + \frac{1}{2}(D_1)^2\right) \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\
 &= \left( \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}^2 \right) \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\
 &= \begin{bmatrix} 4 \\ 5 \\ 5.5 \\ 2 \\ 1.5 \end{bmatrix}
 \end{aligned}$$

Therefore, the ranking from first to last is C, B, A, D, E.

[3 marks]

1 mark for setting up the dominance relationship, including the column matrix of ones.

1 mark for stating the resultant column matrix.

1 mark for providing the correct ranking for all teams.

Note: Consequential on answer to **Question 14a**).

d) Any one of:

- The tie for first place has been split in favour of team C, as team C beat team B in the competition.
- The tie for last place has been split with team E in last place as team D won against team A, who were a higher-ranking team than team E.

[1 mark]

1 mark for drawing a conclusion about either tie and stating how the new results relate to the individual outcomes.

Note: Consequential on answer to **Questions 14b**) and **14c**).

**QUESTION 15 (5 marks)**

- a) Let  $R$  be the resistive force and  $F_R$  the resultant force.

$$F_R = (F \cos(18) - R)\mathbf{i} + (F \sin(18) + N_R - W)\mathbf{j}$$

Considering the horizontal component:

$$F \cos(18) - R = ma$$

$$110 \times \cos(18) - R = 30 \times 2.2$$

$$R = 110 \times \cos(18) - 30 \times 2.2$$

$$= 38.6162\dots \text{ N}$$

[3 marks]

*1 mark for resolving the forces into horizontal and vertical components.*

*1 mark for substituting into the  $f = ma$  formula.*

*1 mark for determining the magnitude of the resistive force .*

- b) Using the vertical component found in Question 15a), or otherwise, gives:

$$100 \times \sin(18) + N_R - W = 0$$

$$N_R = 9.8 \times 30 - 110 \times \sin(18)$$

$$= 260.0081\dots \text{ N}$$

[2 marks]

*1 mark for equating the vertical component to 0.*

*1 mark for calculating the magnitude of the normal reactive force.*

**QUESTION 16 (6 marks)**

a) **Method 1:**

Let  $a = (1+x^2)^{\frac{1}{2}}$  and  $b = x$ .

Thus:

$$a' = \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \times 2x$$

$$= x(1+x^2)^{-\frac{1}{2}}$$

$$b' = 1$$

$$\frac{dv}{dx} = \frac{a'b - b'a}{b^2}$$

$$= \frac{x \times x(1+x^2)^{-\frac{1}{2}} - 1 \times (1+x^2)^{\frac{1}{2}}}{x^2}$$

$$= \frac{(1+x^2)^{-\frac{1}{2}}(x^2 - (1+x^2))}{x^2}$$

$$= \frac{-1}{x^2\sqrt{1+x^2}}$$

$$a = v \times \frac{dv}{dx}$$

$$= \frac{\sqrt{1+x^2}}{x} \times \frac{-1}{x^2\sqrt{1+x^2}}$$

$$= -\frac{1}{x^3}$$

At  $x = 4.5$  m:

$$a = -\frac{1}{4.5^3}$$

$$= -\frac{8}{729} \text{ m s}^{-2} \text{ or } -0.0109\dots \text{ m s}^{-2}$$

[3 marks]

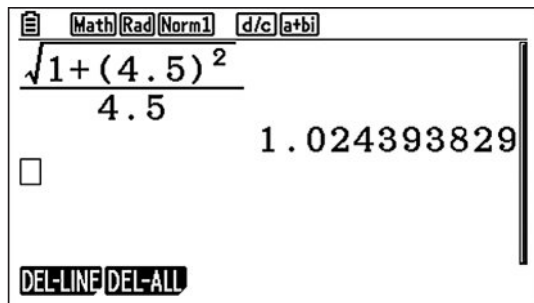
1 mark for determining  $\frac{dv}{dx}$ .

1 mark for using  $a = v \times \frac{dv}{dx}$  to determine a simplified value for  $a$ .

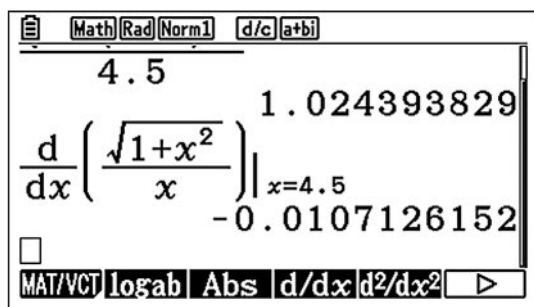
1 mark for determining the acceleration at  $x = 4.5$  m.

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**Method 2:**Using a graphics calculator to solve  $v$  when  $x = 4.5$  m gives:

$$v = 1.02439$$

Using a graphics calculator to solve  $\frac{dv}{dx}$  when  $x = 4.5$  m gives:

$$\frac{dv}{dx} = -0.010713$$

Therefore:

$$\begin{aligned} a &= v \times \frac{dv}{dx} \\ &= 1.02239 \times -0.010713 \\ &= -0.0109 \text{ m s}^{-2} \end{aligned}$$

[3 marks]

1 mark for calculating the value of  $v$ .1 mark for calculating the value of  $\frac{dv}{dx}$ .1 mark for determining the acceleration at  $x = 4.5$  m.

b) 
$$v = \frac{\sqrt{1+x^2}}{x}$$

$$\frac{dx}{dt} = \frac{\sqrt{1+x^2}}{x}$$

Thus:

$$\int \frac{x}{\sqrt{1+x^2}} dx = \int dt$$

Let  $u = 1 + x^2$ ,  $\frac{du}{dx} = 2x$  and  $dx = \frac{du}{2x}$ .

$$\int \frac{x}{\sqrt{u}} \frac{du}{2x} = t + c$$

$$\frac{1}{2} \int u^{-\frac{1}{2}} du = t + c$$

$$\sqrt{u} = t + c$$

$$\sqrt{1+x^2} = t + c$$

At  $x = 2$ ,  $t = 0$  and  $c = \sqrt{5}$ :

$$\sqrt{1+x^2} = t + \sqrt{5}$$

At  $t = 3$ :

$$\sqrt{1+x^2} = 3 + \sqrt{5}$$

$$x = 5.1397 \text{ m}$$

[3 marks]

*1 mark for using differential equations to set up the integral.*

*1 mark for substituting into the integral and finding a relationship between  $x$  and  $t$ .*

*1 mark for providing the correct solution.*



**QUESTION 17 (5 marks)**

Using  $CI_{90} = (397 \text{ g}, 436 \text{ g})$  gives:

$$\begin{aligned}\bar{x} &= \frac{397 + 436}{2} \\ &= 416.5 \text{ g}\end{aligned}$$

$$z_{90} = 1.645$$

Therefore:

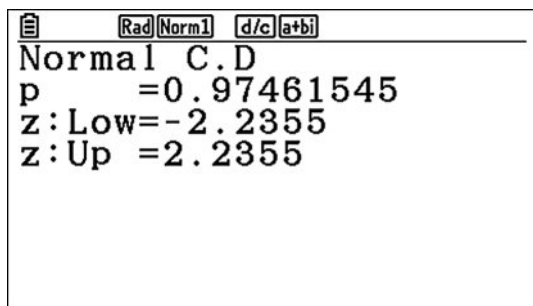
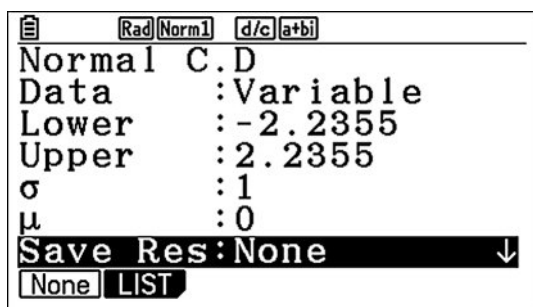
$$\begin{aligned}\text{margin of error} &= z_{90} \times \frac{s}{\sqrt{n}} \\ 416.5 - 397 &= 1.645 \times \frac{s}{\sqrt{n}} \\ \frac{s}{\sqrt{n}} &= \frac{19.5}{1.645}\end{aligned}$$

Let  $CI_x$  be the confidence interval that includes 390 g, where  $x$  is the confidence level.

Given that  $\bar{x} = 416.5 \text{ g}$  and  $\frac{s}{\sqrt{n}} = \frac{19.5}{1.645}$ :

$$\begin{aligned}\text{margin of error} &= z_x \times \frac{s}{\sqrt{n}} \\ 416.5 - 390 &= z_x \times \frac{19.5}{1.645} \\ 26.5 &= z_x \times 11.8541 \\ z_x &= 2.2355\end{aligned}$$

Using a graphics calculator to find the confidence level for  $z_x = 2.2355$  gives:



Thus,  $x = 0.9746$  or 97.46%.

A 97% confidence level will not include 390 g; therefore, 98% is the lowest confidence level that will include 390 g when rounded to a whole number.

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[5 marks]

1 mark for determining the value for  $\frac{s}{\sqrt{n}}$ .

1 mark for applying the  $\frac{s}{\sqrt{n}}$  value to a confidence interval that will contain 390 g.

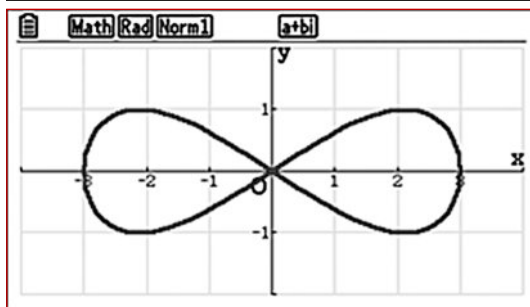
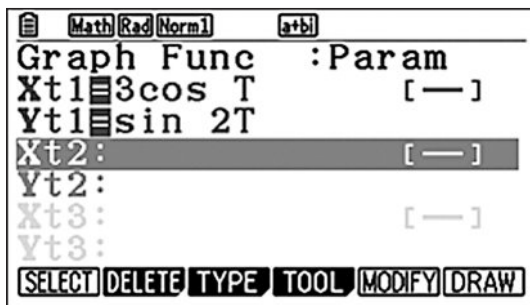
1 mark for determining the value of  $z_x$ .

1 mark for determining the value of  $x$ .

1 mark for providing the correct solution by rounding appropriately.

**QUESTION 18 (5 marks)**

Letting  $a = 3$  to find the initial shape of the function gives:



When  $r(\theta) = (a\cos(\theta))i + (\sin(2\theta))j$ :

$$x = a\cos(\theta)$$

$$y = \sin(2\theta)$$

$$= 2\sin(\theta)\cos(\theta)$$

$$x^2 = a^2 \cos^2(\theta)$$

$$= a^2(1 - \sin^2(\theta))$$

Therefore,  $\cos^2(\theta) = \frac{x^2}{a^2}$  and  $\sin^2(\theta) = 1 - \frac{x^2}{a^2}$ .

Substituting into  $y^2 = 4\sin^2(\theta)\cos^2(\theta)$  gives:

$$y^2 = \frac{4x^2}{a^2} \left(1 - \frac{x^2}{a^2}\right)$$

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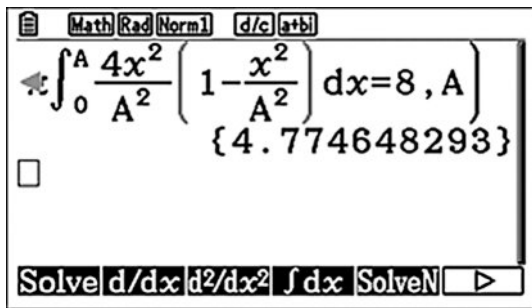
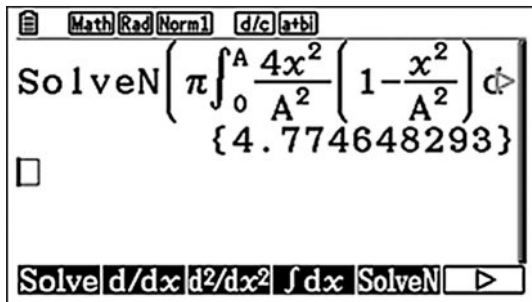
(continued)

Finding the expression for the volume:

$$V = \int_0^a \pi y^2 dx$$

$$= \pi \int_0^a \frac{4x^2}{a^2} \left(1 - \frac{x^2}{a^2}\right) dx = 8$$

Using a graphics calculator to solve for  $a$  gives:



[5 marks]

1 mark for using trigonometric identities to simplify the  $\mathbf{j}$  component of the vector.  
 1 mark for using the Pythagorean identity to prepare the  $\mathbf{i}$  component of the vector for substitution.

1 mark for determining the Cartesian form of the vector equation.  
 1 mark for using the volumes of revolution formula to find an appropriate integral.  
 1 mark for providing the correct value of  $a$ .

**QUESTION 19 (6 marks)**

Motion of the ball:

Finding the velocity of the ball gives:

$$a = -g\mathbf{j}$$

$$v = -gt\mathbf{j} + v_0$$

$$v_0 = u \cos(45)\mathbf{i} + u \sin(45)\mathbf{j}$$

$$= \frac{u}{\sqrt{2}}\mathbf{i} + \frac{u}{\sqrt{2}}\mathbf{j}$$

$$v = \frac{u}{\sqrt{2}}\mathbf{i} + \left(\frac{u}{\sqrt{2}} - gt\right)\mathbf{j}$$

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(continued)

Finding the displacement of the ball gives:

$$x = \frac{ut}{\sqrt{2}}\mathbf{i} + \left( \frac{ut}{\sqrt{2}} - \frac{gt^2}{2} \right)\mathbf{j} + x_0$$

$$x_0 = 1.8\mathbf{j}$$

$$x = \frac{ut}{\sqrt{2}}\mathbf{i} + \left( 1.8 + \frac{ut}{\sqrt{2}} - \frac{gt^2}{2} \right)\mathbf{j}$$

As the ball passes through the point (15, 2.5), simultaneous equations can be formed.

$$\frac{ut}{\sqrt{2}} = 15 \quad (1)$$

$$1.8 + \frac{ut}{\sqrt{2}} - \frac{gt^2}{2} = 2.5 \quad (2)$$

Substituting (1) into (2) to solve for  $t$  gives:

$$1.8 + 15 - \frac{gt^2}{2} = 2.5$$

$$t = 1.708 \text{ s}$$

Substituting  $t = 1.708$  into (1) gives:

$$\frac{u \times 1.708}{\sqrt{2}} = 15$$

$$u = 12.4176 \text{ m s}^{-1}$$

Motion of Barry:

Barry travels from (20, 5) to (15, 2.5) before catching the ball. Therefore, the distance travelled is:

$$\begin{aligned} d &= \sqrt{(15 - 20)^2 + (2.5 - 5)^2} \\ &= 5.59 \text{ m} \end{aligned}$$

Finding the time taken for Barry to move halfway down the escalator gives:

$$\begin{aligned} t &= \frac{d}{s} \\ &= \frac{5.59}{0.5} \\ &= 11.18 \text{ s} \end{aligned}$$

Chelsea should throw the ball 11.18 – 1.708 seconds after Barry steps onto the escalator.

Therefore, Chelsea should throw the ball at a speed of 12.4176 m s<sup>-1</sup> and at a time of 9.47 seconds for Barry to catch it when halfway down the escalator.

[6 marks]

*1 mark for determining a function that represents the ball's velocity.*

*1 mark for determining a function that represents the ball's displacement.*

*1 mark for setting up two equations for  $u$  and  $t$  that align with the coordinate of impact.*

*1 mark for determining the values of  $u$  and  $t$  at the moment of impact.*

*1 mark for determining the time taken for Barry to move halfway down the escalator.*

*1 mark for providing the correct solution.*