Neap

Trial Examination 2022

Question and Response Booklet

QCE Specialist Mathematics Units 3&4

Paper 2 – Technology-active

Student's Name: ______________________

Teacher's Name: ______________________

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator **permitted**.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 8 short response questions

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has eight questions and is worth 55 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (6 marks)

a) Use Simpson's rule with four strips to approximate the definite integral $\int_0^{\frac{\pi}{2}} \cos\left(x^2\right)dx$.

Complete the table below as part of your response. *[4 marks]*

x	$y = \cos\left(x^2\right)$
x_0	$y_0 =$
x_1	$y_1 =$
x_2	$y_2 =$
x_3	$y_3 =$
x_4	$y_4 =$

b) Evaluate the reasonableness of your result from Question 11a) by comparing your result with the solution of the original integral from your graphics calculator. *[2 marks]*

QUESTION 12 (5 marks)

Let $u = 3 - 2i$ and $v = 5 + 2i$.

a) Write u and v in polar form. *[2 marks]*

b) Evaluate $\dfrac{u^2}{v^3}$. *[3 marks]*

QUESTION 13 (8 marks)

Bubbles of gas can be produced deep in the ocean by a range of geological, chemical or biological processes. As these bubbles rise towards the surface of the water, they increase in volume at a rate that is inversely proportional to the water pressure, which decreases as depth decreases.

The relationship between depth, z (in metres), and pressure, p (in Pascals), is

$$\frac{dp}{dz} = -\rho g,$$

where the acceleration due to gravity (g) is 9.8 m s^{-2} and the density of gas (ρ) at pressure p is $\frac{pM}{RT}$.

The molecular mass of the gas (M) and the ideal gas constant (R) are constants. Assume the temperature (T), measured in Kelvin (K), is constant.

a) Take V_1 to be the volume of a bubble at depth z_1 and V_2 to be the volume of a bubble at depth z_2.

Show that $V_2 = V_1 e^{\left[\frac{Mg}{RT}(z_2 - z_1)\right]}$. *[5 marks]*

b) Determine the change in depth that is required for the radius of a bubble of oxygen gas ($M = 32$) to double. Assume that $R = 8.314$ and $T = 300$ K ($\approx$ 23°C). *[3 marks]*

QUESTION 14 (6 marks)

The data in the table shows the age and number of female lizards in a population of lizards. The data was recorded annually over four years.

Year recorded	Number of female lizards		
	< 1 year old	1 year old	2 years old
2005	75	66	57
2006	120	56	54
2007	124	89	45
2008	151	92	73

Develop a mathematical model that can be used to estimate future populations. State any assumptions made and evaluate the reasonableness of your model.

QUESTION 15 (8 marks)

A golf ball is hit from a golf tee towards a hole on a wind-free day. Its position vector is given by

$$\boldsymbol{r}(t) = 75t\,\hat{\boldsymbol{i}} + 2.8t\,\hat{\boldsymbol{j}} + 50\sin(0.9t)\hat{\boldsymbol{k}},$$

where displacement, $\boldsymbol{r}$, is measured in metres and the time, t, is measured in seconds. The unit vectors $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$ are horizontal; $\hat{\boldsymbol{i}}$ is aligned north. $\hat{\boldsymbol{k}}$ is a unit vector in the vertical direction.

The hole is 250 m from the golf tee, 25° east of north. The landscape rises at a constant slope of 2° between the tee and the hole.

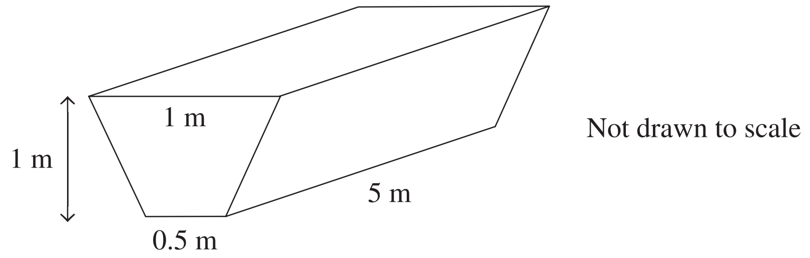
a) Calculate the initial speed of the golf ball as it leaves the tee. *[2 marks]*

b) Determine the equation of the plane for the landscape between the tee and the hole. *[3 marks]*

c) Calculate the distance between the landing position of the golf ball and the hole. *[3 marks]*

QUESTION 16 (8 marks)

A water trough for animals is 5 m long with a trapezoidal side, as shown in the diagram.

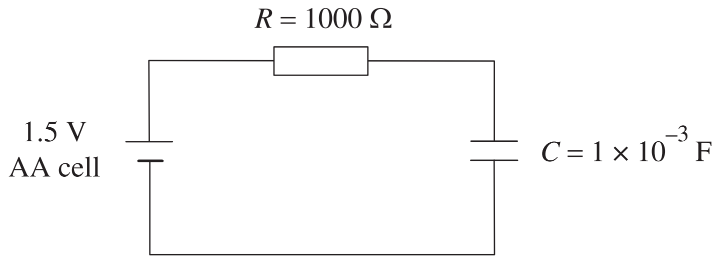


The water level of the trough is reduced when animals drink from it and through evaporation. When the depth of the water in the trough is 5 cm, an automatic pump is triggered. When triggered, the pump adds water to the trough at a rate of 5 L per minute until it is full again.

Determine the rate at which the depth of the water is increasing when the tank is half full. Assume that $t = 0$ minutes when the pump starts.

QUESTION 17 (7 marks)

The diagram shows a single AA cell connected in series with a resistor ($R = 1000\ \Omega$) and a capacitor ($C = 1 \times 10^{-3}$ F) in a series RC circuit.



The equation describing the relationship between the charge stored in the capacitor, Q, and the rate of change of the charge stored in the capacitor, $\frac{dQ}{dt}$, is

$$V - R\frac{dQ}{dt} - \frac{Q}{C} = 0,$$

where V is the battery voltage in volts and R is the value of the resistor in Ohms.

a) Show that this differential equation can be written as $Q = CV\left(1 - e^{-\frac{t}{RC}}\right)$, assuming that the initial charge in the capacitor is zero. *[3 marks]*

b) Determine the value of the charge in the capacitor when $t = 0.004$ s. *[2 marks]*

c) Determine the rate of change of the charge, $\frac{dQ}{dt}$, when $Q = 5 \times 10^{-5}$ C. *[2 marks]*

QUESTION 18 (7 marks)

A company sells sugar sachets that are commonly used in tea and coffee. The mass of sugar in the sachets was found to be normally distributed, with a known mean of μ and standard deviation of ρ.

The company sells the sugar sachets in boxes of 50. A sample of 30 boxes of sugar sachets is selected and the average total mass of the sachets in each box is calculated. The mean of the averages is 3.01 g, and the standard deviation of the sample is 0.17 g.

a) Determine μ and σ. *[1 mark]*

b) The total mass of the sugar sachets in a particular box is 157 g.

Determine the 95% confidence interval for the mass of each sugar sachet in this box. *[3 marks]*

c) Determine the probability that the total mass of sugar sachets in one box is less than 150 g. *[3 marks]*

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula Booklet

QCE Specialist Mathematics Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + c$
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

Calculus		
chain rule	If $h(x)=f\left(g(x)\right)$ then $h'(x)=f'\left(g(x)\right)g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\dfrac{dy}{dx}=\dfrac{dy}{du}\times\dfrac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\dfrac{d}{dx}(uv)=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$
quotient rule	If $h(x)=\dfrac{f(x)}{g(x)}$ then $h'(x)=\dfrac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	$\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$	$\int u\dfrac{dv}{dx}dx=uv-\int v\dfrac{du}{dx}dx$
volume of a solid of revolution	about the x-axis	$V=\pi\int_a^b\left[f(x)\right]^2dx$
	about the y-axis	$V=\pi\int_a^b\left[f(y)\right]^2dy$
Simpson's rule	$\int_a^b f(x)dx\approx\dfrac{w}{3}\left[f(x_0)+4\left[f(x_1)+f(x_3)+\ldots\right]+2\left[f(x_2)+f(x_4)+\ldots\right]+f(x_n)\right]$	
simple harmonic motion	If $\dfrac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin(\omega t+\alpha)$ or $x=A\cos(\omega t+\beta)$	
	$v^2=\omega^2\left(A^2-x^2\right)$	$T=\dfrac{2\pi}{\omega}$ $f=\dfrac{1}{T}$
acceleration	$a=\dfrac{dv}{dt}=\dfrac{d^2x}{dt^2}=v\dfrac{dv}{dx}=\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z=x+yi=r\left(\cos(\theta)+i\sin(\theta)\right)=r\,\mathrm{cis}(\theta)$
modulus	$\lvert z\rvert=r=\sqrt{x^2+y^2}$
argument	$\arg(z)=\theta,\ \tan(\theta)=\dfrac{y}{x},-\pi<\theta\le\pi$
product	$z_1z_2=r_1r_2\,\mathrm{cis}\left(\theta_1+\theta_2\right)$
quotient	$\dfrac{z_1}{z_2}=\dfrac{r_1}{r_2}\mathrm{cis}\left(\theta_1-\theta_2\right)$
De Moivre's theorem	$z^n=r^n\mathrm{cis}(n\theta)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
permutation	${}^nP_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$ $\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$ $\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$ $\cos(A)\sin(B) = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$

Vectors and matrices		
magnitude	$\lvert \boldsymbol{a} \rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$	
scalar (dot) product	$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \cos(\theta)$	
	$\boldsymbol{a} \cdot \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$	
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$	
Cartesian equation of a line	$\dfrac{x - a_1}{d_1} = \dfrac{y - a_2}{d_2} = \dfrac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \sin(\theta) \hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert \boldsymbol{a} \rvert \cos(\theta) \hat{\boldsymbol{b}} = \left(\boldsymbol{a} \cdot \hat{\boldsymbol{b}} \right) \hat{\boldsymbol{b}}$	
vector equation of a plane	$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\mathbf{A}) = ad - bc$	
multiplicative inverse matrix	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, $\det(\mathbf{A}) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ m s}^{-2}$