Neap

Trial Examination 2022

Suggested Solutions

QCE Specialist Mathematics Units 3&4

Paper 2 – Technology-active

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.	●	○	○	○
2.	○	●	○	○
3.	○	○	○	●
4.	○	○	●	○
5.	○	○	○	●
6.	○	○	●	○
7.	●	○	○	○
8.	●	○	○	○
9.	○	○	●	○
10.	○	○	●	○

QUESTION 1 A

A is correct. Using a graphics calculator gives $\mathbf{A}^{-1}\mathbf{B}$ as:

```
[A]^-1
                 [-24  18   5]
                 [ 20 -15  -4]
                 [ -5   4   1]
[A]^-1*[B]
              [ 24  -76  205]
              [-20   65 -169]
              [  6  -16   44]
```

B is incorrect. This option shows **AB**.

C is incorrect. This option shows $\mathbf{BA}^{-1}$.

D is incorrect. This option shows $\mathbf{AB}^{-1}$.

QUESTION 2 B

Converting to polar form using a graphics calculator and applying De Moivre's theorem gives

$\left(7.56\text{cis}\left(-0.48\right)\right)^{0.2} = 1.50\text{cis}\left(-0.096\right)$.

```
R▸Pr(6.7, -3.5)
                 7.559100476
R▸Pθ(6.7, -3.5)
               -0.4813972241
```

Converting back into Cartesian form using a graphics calculator gives $1.49 - 0.14i$.

```
P▸Rx(1.5, -.096)
                 1.493093307
P▸Ry(1.5, -0.096)
               -0.1437789179
```

QUESTION 3 D

The 95% confidence interval is calculated using $\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}},\ \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right)$, where $\sigma = 0.9$, $\bar{x} = 3.2$ and $n = 100$.

$$
\begin{aligned}
CI &= \left(3.2 - 1.96\frac{0.9}{\sqrt{100}},\ 3.2 + 1.96\frac{0.9}{\sqrt{100}}\right) \\
&= (3.02,\ 3.38)
\end{aligned}
$$

QUESTION 4 C

Defining $\hat{\boldsymbol{i}}$ so that it is parallel with the 9.4 N force vector allows the vector sum to be written as:

$$
\begin{aligned}
F_N &= 9.4\hat{\boldsymbol{i}} + 6.3\cos(37)\hat{\boldsymbol{i}} + 6.3\sin(37)\hat{\boldsymbol{j}} \\
&= 14.43\hat{\boldsymbol{i}} + 3.79\hat{\boldsymbol{j}}
\end{aligned}
$$

$$|F_N| = \sqrt{14.43^2 + 3.79^2}$$

$$|F_N| \approx 14.9 \text{ N}$$

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QUESTION 5 D

The arrival of 20 buses per hour, or 20 buses per 60 minutes, gives a mean time between buses of $\frac{60}{20} = 3$ minutes. Therefore, $\lambda = \frac{1}{3}$.

$$\begin{aligned} F(x \le 5) &= \int_0^5 \frac{1}{3} e^{\frac{-x}{3}} \, dx \\ &= \left[-e^{\frac{-x}{3}} \right]_0^5 \\ &= -e^{\frac{-5}{3}} + 1 \\ &= -0.19 + 1 \\ &= 0.81 \end{aligned}$$

QUESTION 6 C

$\overline{\overline{x}} = 375$

$\sigma = 52$

$n = 50$

$$\begin{aligned} \Pr\left(\overline{\overline{X}} > 390\right) &= \Pr\left(Z > \frac{390 - 375}{\frac{52}{\sqrt{50}}} \right) \\ &= \Pr(Z > 2.04) \\ &= 0.02 \end{aligned}$$

QUESTION 7 A

Converting to polar form using a graphics calculator gives:

```
P▸Rx(5,135)
                 -3.535533906
P▸Ry(5,135)
                  3.535533906
P▸Rx(2.2,65)
                 0.9297601758
P▸Ry(2.2,65)
                  1.993877131
```

$\boldsymbol{a} = -3.54\hat{\boldsymbol{i}} + 3.54\hat{\boldsymbol{j}}$ and $\boldsymbol{b} = 0.93\hat{\boldsymbol{i}} + 1.99\hat{\boldsymbol{j}}$.

$$\begin{aligned} \boldsymbol{a} \times \boldsymbol{b} &= \left(-3.54\hat{\boldsymbol{i}} \times 1.99\hat{\boldsymbol{j}}\right) + \left(3.54\hat{\boldsymbol{j}} \times 0.93\hat{\boldsymbol{i}}\right) \\ &= -7.04\hat{\boldsymbol{k}} - 3.29\hat{\boldsymbol{k}} \\ &= -10.33\hat{\boldsymbol{k}} \end{aligned}$$

QUESTION 8 A

$$\int_0^{\frac{1}{a}} 3e^{-ax}\,dx = 7$$

$$\left[\frac{3e^{-ax}}{-a}\right]_0^{\frac{1}{a}} = 7$$

$$\left(\frac{3e^{-1}}{-a} - \frac{3e^{0}}{-a}\right) = 7$$

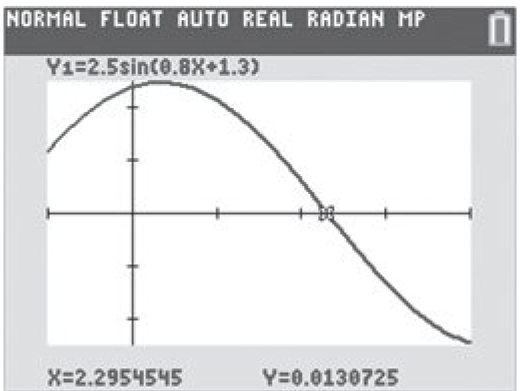
$$3e^{-1} - 3e^{0} = -7a$$

$$a = \frac{3\left(e^{-1} - 1\right)}{-7}$$

$$= 0.27$$

QUESTION 9 C

C is correct. Maximum speed is reached when $x = 0$; that is, when $0.8t + 1.3 = 0$. Using a graphics calculator, the first point at which $x = 0$, given that $t \geq 0$ s, occurs at $t = 2.3$ s.



A is incorrect. This option may be reached as a simple algebraic solution to $0.8t + 1.3 = 0$, but it gives a value for $t < 0$; that is, before the start of the particle's motion and outside the boundaries of the question.

B and **D** are incorrect. These options refer to points where the particle is at the extremities of motion and, therefore, has maximum acceleration but zero speed.

QUESTION 10 C

$$\frac{dP}{dt} = 0.03P$$

$$\frac{dP}{P} = 0.03dt$$

$$\ln(P) = 0.03t + c$$

At $t = 0$, $P = 12$.

$$c = \ln(12)$$

$$P = 12e^{0.03t}$$

Evaluating at $t = 76$ years gives $P \approx 117$ goats.

SECTION 2

QUESTION 11 (6 marks)

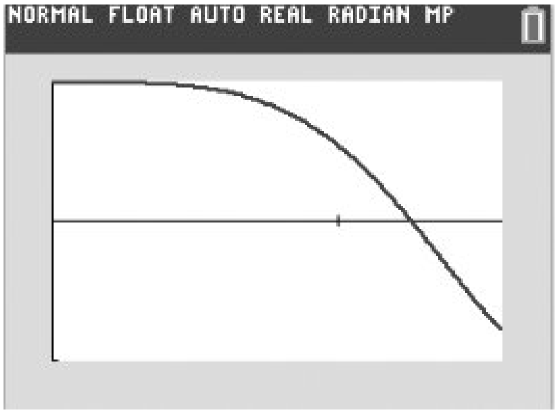
a)

x	$y = \cos(x^2)$
x_0	$y_0 = 1$
x_1	$y_1 = 0.988$
x_2	$y_2 = 0.815$
x_3	$y_3 = 0.1819$
x_4	$y_4 = -0.7812$

$$\text{area} = \frac{\pi}{24}\left[1 + 4(0.988 + 0.1819) + 2(0.815) - 0.7812\right]$$
$$\approx 0.855$$

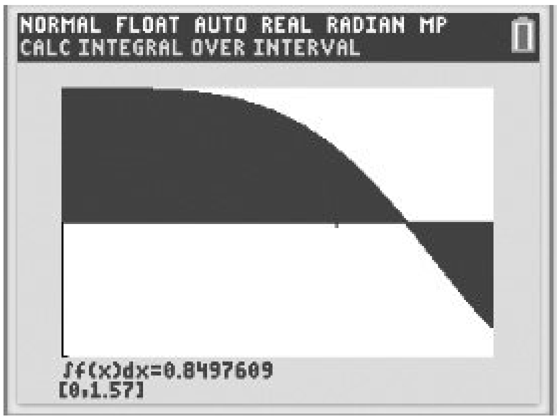
[4 marks]
2 marks for completing the table. Note: Award 1 mark for 3–4 correct table entries.
1 mark for identifying the equation for calculating the area. Note: This may be implied by subsequent working.
1 mark for calculating the area.

b) Using a graphics calculator:



Using Simpson's rule may not provide a valid answer as the line crosses the x-axis.

Using a graphics calculator gives the definite integral as ≈ 0.85. The result obtained using Simpson's rule and the result obtained using a graphics calculator are close. Therefore, the result from Question 11a) is reasonable.



[2 marks]

1 mark for providing the correct solution using a graphics calculator.

1 mark for commenting on whether the result obtained in Question 11a) is reasonable or not reasonable.

QUESTION 12 (5 marks)

a)
$$|u| = \sqrt{3^2 + 2^2}$$
$$= \sqrt{13}$$
$$\approx 3.61$$
$$\text{Arg}(u) = \tan^{-1}\left(-\frac{2}{3}\right)$$
$$= -33.7°$$
$$u = 3.61\,\text{cis}(-33.7°)$$
$$|v| = \sqrt{5^2 + 2^2}$$
$$= \sqrt{29}$$
$$\approx 5.39$$
$$\text{Arg}(v) = \tan^{-1}\left(\frac{2}{5}\right)$$
$$= 21.8°$$
$$v = 5.39\,\text{cis}(21.8°)$$

[2 marks]
1 mark for writing u in polar form.
1 mark for writing v in polar form.

b)
$$u^2 = 3.61^2\,\text{cis}(2 \times -33.7°)$$
$$= 13.03\,\text{cis}(-67.4°)$$
$$v^3 = 5.39^3\,\text{cis}(3 \times 21.8°)$$
$$= 156.6\,\text{cis}(65.4°)$$
$$\frac{u^2}{v^3} = \frac{13.03}{156.6}\text{cis}(-67.4° - 65.4°)$$
$$= 0.083\text{cis}(-132.8°)$$

[3 marks]
1 mark for calculating u^2.
1 mark for calculating v^3.
1 mark for calculating $\frac{u^2}{v^3}$.

Note: Accept solutions in either Cartesian or polar form.

QUESTION 13 (8 marks)

a)

$$\frac{dp}{dz} = -\rho g$$

$$= -\frac{pMg}{RT}$$

$$\frac{dp}{p} = -\frac{Mg}{RT}dz$$

$$\int_{p_1}^{p_2} \frac{dp}{p} = \int_{z_1}^{z_2} \frac{-Mg}{RT} dz$$

$$\left[\ln(p)\right]_{p_1}^{p_2} = \left[-\frac{Mgz}{RT}\right]_{z_1}^{z_2}$$

$$\ln(p_2) - \ln(p_1) = -\frac{Mg}{RT}(z_2 - z_1)$$

$$\frac{p_2}{p_1} = e^{\left[-\frac{Mg}{RT}(z_2 - z_1)\right]}$$

As pressure, p, and volume, V, are inversely proportional, $\frac{p_2}{p_1} = \frac{V_1}{V_2}$.

$$\frac{V_1}{V_2} = e^{\left[-\frac{Mg}{RT}(z_2 - z_1)\right]}$$

$$\frac{V_2}{V_1} = e^{\left[\frac{Mg}{RT}(z_2 - z_1)\right]}$$

$$V_2 = V_1 e^{\left[\frac{Mg}{RT}(z_2 - z_1)\right]}$$

[5 marks]

1 mark for identifying the correct differential equation.

1 mark for setting up the integration from the differential equation.

1 mark for evaluating the integral.

1 mark for creating the equation describing the relationship between p_1 and p_2.

1 mark for creating the equation describing the relationship between V_1 and V_2.

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b) If the radius of a spherical bubble doubles, then the volume increases by a factor of 8.

$V = \frac{4}{3}\pi r^3$; therefore, $V_2 = 8V_1$.

$$8 = e^{\left[\frac{Mg}{RT}(z_2 - z_1)\right]}$$

$$\frac{Mg}{RT}(z_2 - z_1) = \ln(8)$$

$$z_2 - z_1 = \frac{\ln(8)RT}{Mg}$$

$$z_2 - z_1 = \frac{\ln(8) \times 8.314 \times 300}{32 \times 9.8}$$

$$z_2 - z_1 \approx 16.5 \text{ m}$$

[3 marks]

1 mark for substituting $V_2 = 8V_1$.

1 mark for finding the equation that identifies the difference $z_2 - z_1$.

1 mark for calculating the difference in depth.

QUESTION 14 (6 marks)

$$\mathbf{L} = \begin{bmatrix} B_1 & B_2 & B_3 \\ S_1 & 0 & 0 \\ 0 & S_2 & 0 \end{bmatrix}, \mathbf{P}_0 = \begin{bmatrix} 75 \\ 66 \\ 57 \end{bmatrix}$$

$$\mathbf{P}_1 = \mathbf{LP}_0$$

$$\begin{bmatrix} 120 \\ 56 \\ 54 \end{bmatrix} = \begin{bmatrix} B_1 & B_2 & B_3 \\ S_1 & 0 & 0 \\ 0 & S_2 & 0 \end{bmatrix}\begin{bmatrix} 75 \\ 66 \\ 57 \end{bmatrix}$$

$$\mathbf{P}_2 = \mathbf{LP}_1$$

$$\begin{bmatrix} 124 \\ 89 \\ 45 \end{bmatrix} = \begin{bmatrix} B_1 & B_2 & B_3 \\ S_1 & 0 & 0 \\ 0 & S_2 & 0 \end{bmatrix}\begin{bmatrix} 120 \\ 56 \\ 54 \end{bmatrix}$$

$$\mathbf{P}_3 = \mathbf{LP}_2$$

$$\begin{bmatrix} 151 \\ 92 \\ 73 \end{bmatrix} = \begin{bmatrix} B_1 & B_2 & B_3 \\ S_1 & 0 & 0 \\ 0 & S_2 & 0 \end{bmatrix}\begin{bmatrix} 124 \\ 89 \\ 45 \end{bmatrix}$$

$120 = 75B_1 + 66B_2 + 57B_3$ (1)

$124 = 120B_1 + 56B_2 + 54B_3$ (2)

$151 = 124B_1 + 89B_2 + 45B_3$ (3)

$56 = 75S_1$

$S_1 = 0.75$

$54 = 66S_2$

$S_2 = 0.82$

Equations (1), (2) and (3) can be solved as a series of simultaneous equations.

$$\begin{bmatrix}120\\124\\151\end{bmatrix}=\begin{bmatrix}75&66&57\\120&56&54\\124&89&45\end{bmatrix}\begin{bmatrix}B_1\\B_2\\B_3\end{bmatrix}$$

$$\begin{bmatrix}75&66&57\\120&56&54\\124&89&45\end{bmatrix}^{-1}\begin{bmatrix}120\\124\\151\end{bmatrix}=\begin{bmatrix}B_1\\B_2\\B_3\end{bmatrix}$$

$$\begin{bmatrix}B_1\\B_2\\B_3\end{bmatrix}=\begin{bmatrix}0.33\\0.94\\0.58\end{bmatrix}$$

Therefore, the Leslie matrix that can be used to model future population growth is:

$$\mathbf{L}=\begin{bmatrix}0.33&0.94&0.58\\0.75&0&0\\0&0.82&0\end{bmatrix},\ \mathbf{P}_0=\begin{bmatrix}75\\66\\57\end{bmatrix}$$

Evaluating the reasonableness of the model:

$$\mathbf{P}_1=\mathbf{LP}_0=\begin{bmatrix}120\\56\\54\end{bmatrix}$$

$$\mathbf{P}_2=\mathbf{L}^2\mathbf{P}_0=\begin{bmatrix}123\\90\\46\end{bmatrix}$$

$$\mathbf{P}_3=\mathbf{L}^3\mathbf{P}_0=\begin{bmatrix}152\\93\\74\end{bmatrix}$$

This model is very accurate over the four years for which data has been gathered. The use of a Leslie matrix assumes that the population growth rate model is steady and unchanging year to year.

[6 marks]

1 mark for establishing the three simultaneous equations.

1 mark for calculating S_1 and S_2. Note: Allow follow-through errors.

1 mark for calculating B_1, B_2 and B_3.

1 mark for writing the Leslie matrix.

1 mark for evaluating the reasonableness of the model. Note: Accept checking P_3 only.

1 mark for making a statement about the assumptions and accuracy of the model.

QUESTION 15 (8 marks)

a) $\boldsymbol{r}(t) = 75t\,\hat{\boldsymbol{i}} + 2.8t\,\hat{\boldsymbol{j}} + 50\cos(0.9t)\hat{\boldsymbol{k}}$

At $t = 0$, $\boldsymbol{r}(0) = 75\hat{\boldsymbol{i}} + 2.8\hat{\boldsymbol{j}} + 45\hat{\boldsymbol{k}}$

$$\begin{aligned}\text{magnitude of } \boldsymbol{r}(0) &= \sqrt{75^2 + 2.8^2 + 45^2} \\ &= 87.5 \text{ m s}^{-1}\end{aligned}$$

[2 marks]
1 mark for calculating r(0).
1 mark for calculating the magnitude of r(0).

b) Two displacement vectors are known: the golf tee, $\boldsymbol{r}_g = (0, 0, 0)$, and the hole, $\boldsymbol{r}_h = \begin{pmatrix} 250 \text{ m} \\ 25° \\ 2° \end{pmatrix}$.

In Cartesian coordinates, the displacement vector to the hole can be written as $\boldsymbol{r}_h = (226.4, 105.6, 8.7)$.

A possible normal vector to the plane is $\boldsymbol{n} = \begin{pmatrix} 250 \text{ m} \\ 25° \\ 92° \end{pmatrix}$ or $\tilde{\boldsymbol{n}} = (-7.9, -3.7, 249.8)$ in Cartesian coordinates.

(Note: The magnitude of this vector is arbitrary, and could equally be set to 1, or any other value.)

The equation of the plane can then be calculated using $\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r}_o)$, where $\boldsymbol{r} = (x, y, z)$ and $\boldsymbol{r}_o$ is any known displacement vector on the plane. In this instance, the easiest point to use is $\boldsymbol{r}_g = (0, 0, 0)$.

The equation of the plane is:

$$\begin{aligned}(-7.9\hat{\boldsymbol{i}} - 3.7\hat{\boldsymbol{j}} + 249.8\hat{\boldsymbol{k}}) \cdot ((x-0)\hat{\boldsymbol{i}} + (y-0)\hat{\boldsymbol{j}} + (z-0)\hat{\boldsymbol{k}}) &= 0 \\ -7.93x - 3.7y + 249.8z &= 0\end{aligned}$$

[3 marks]
1 mark for calculating $\boldsymbol{r}_h$ *in Cartesian coordinates.*
1 mark for identifying a possible normal vector.
1 mark for identifying the equation of the plane.

c) When the ball initially lands, the displacement vector of the ball will be coincident with the plane.

Written in parametric form, the displacement vector of the ball is:

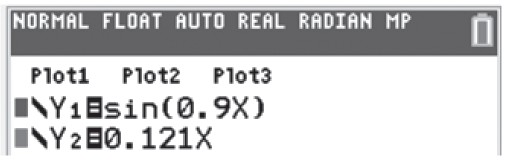
$x = 75t,\ y = 2.8t,\ z = 50\sin(0.9t)$

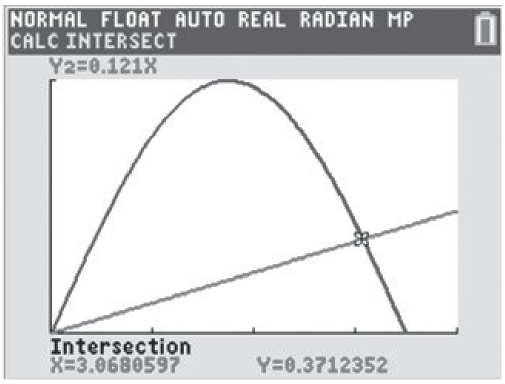
The intersection between this vector and the plane can be found by substituting the parametric equations into the equation for the plane, giving:

$$-7.93 \times 75t - 3.7 \times 2.8t + 249.8 \times 50\sin(0.9t) = 0$$

$$\sin(0.9t) = 0.121t$$

This equation can be solved using a graphics calculator to give $t = 3.07$ s.





Note: The x-axis represents time.

The position of the ball at $t = 3.07$ s can be found by substituting in $t = 3.07$ s into the position vector.

$$\boldsymbol{r}(t = 3.07\text{ s}) = 75 \times 3.07\hat{\boldsymbol{i}} + 2.8 \times 3.07\hat{\boldsymbol{j}} + 50\sin(0.9 \times 3.07)\hat{\boldsymbol{k}}$$

$$\boldsymbol{r}(t = 3.07\text{ s}) = 230\hat{\boldsymbol{i}} + 8.6\hat{\boldsymbol{j}} + 18.5\hat{\boldsymbol{k}}$$

The displacement vector of the hole from 15b) is $\boldsymbol{r}_h = 226.4\hat{\boldsymbol{i}} + 105.5\hat{\boldsymbol{j}} + 8.7\hat{\boldsymbol{k}}$.

Distance between the hole and the ball when it lands:

$$\left|\boldsymbol{r}(t = 3.07) - \boldsymbol{r}_h\right| = \sqrt{(230 - 226.4)^2 + (8.6 - 105.6)^2 + (18.5 - 8.7)^2}$$

$$\text{distance} = 97.6\text{ m}$$

[3 marks]

1 mark for identifying the displacement vector in parametric form.
1 mark for calculating the time and position of the ball when it lands.
1 mark for calculating the distance between the ball and the hole.
Note: Consequential on answer to ***Question 15b).***

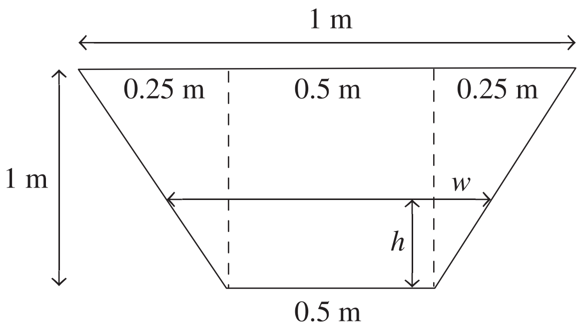
QUESTION 16 (8 marks)

For consistency, all measurements should be in metres for length and cubic metres for volume.

$\frac{dV}{dt} = 0.005 \text{ m}^3/\text{min}$

$V = 0.005t + C$

C is the volume of water in the tank at $t = 0$, when the water has a depth of 0.05 m.



$\frac{w}{h} = \frac{0.25}{1}$ (similar triangles)

$w = 0.25h$

$$\begin{aligned}\text{area of a trapezium} &= \frac{a+b}{2} \times h \\ &= \frac{(0.5 + 2w) + 0.5}{2} \times h \\ &= \frac{1 + 2 \times 0.25h}{2} \times h \\ &= \frac{1}{4}h^2 + \frac{1}{2}h\end{aligned}$$

volume of water at depth h (trapezoidal prism) $= \frac{1}{4}(h^2 + 2h) \times 5$ m

$\therefore$ initial volume (at $h = 0.05$ m), $C = 0.128 \text{ m}^3$

$V = 0.005t + 0.128 \text{ m}^3$

To find $\frac{dh}{dt}$ when the tank is half full:

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$$\begin{aligned}\frac{dV}{dh} &= \frac{d}{dh}\left[\frac{1}{4}(h^2 + 2h) \times 5 \text{ m}\right] \\ &= \frac{5}{2}(h+1)\end{aligned}$$

So, $\frac{dh}{dV} = \frac{2}{5(h+1)}$ and $\frac{dV}{dt} = 0.005$.

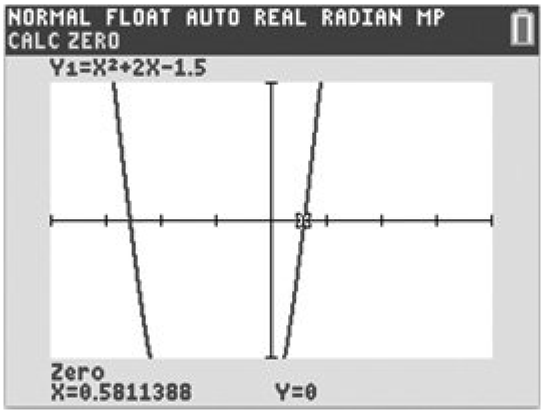
Thus, $\frac{dh}{dt} = \frac{2}{5(h+1)} \times 0.005$.

To find $\frac{dh}{dt}$ when the tank is half full, it is necessary to find h when the tank is half full.

When full, the volume of the tank is 3.75 m^3. Therefore, half the volume is 1.875 m^3.

$$V = \frac{1}{4}\left(h^2 + 2h\right) \times 5 \text{ m}$$

Solving for h when $V = 1.875$ using a graphics calculator gives $h = 0.581$ m.



$$1.875 = \frac{1}{4}(h^2 + 2h) \times 5$$

$$1.5 = h^2 + 2h$$

$$h^2 + 2h - 1.5 = 0$$

$$\frac{dh}{dt} = \frac{2}{5\left(0.581 + 1\right)} \times 0.005$$

$= 0.0013$ m/min or approximately 1.3 mm/min

[8 marks]

1 mark for integrating the differential equation.

1 mark for finding the area in terms of height, h.

1 mark for finding the volume in terms of height, h.

1 mark for finding V(t).

1 mark for identifying the differential chain $\frac{dh}{dt}$.

1 mark for determining $\frac{dh}{dt}$.

1 mark for calculating the volume and height when the trough is half full.

1 mark for providing the correct solution.

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QUESTION 17 (7 marks)

a)
$$\frac{dQ}{dt}=\frac{VC-Q}{CR}$$
$$\frac{dQ}{VC-Q}=\frac{dt}{CR}$$
$$-\ln(VC-Q)=\frac{t}{CR}+k$$
$$\ln(VC-Q)=-\frac{t}{CR}+k$$
$$VC-Q=e^{-\frac{t}{CR}}\cdot e^{k}$$

At $t=0$, $Q=0$.
$$VC=e^{k}$$
$$VC-Q=VCe^{-\frac{t}{CR}}$$
$$Q=VC-VCe^{-\frac{t}{CR}}$$
$$=CV\left(1-e^{-\frac{t}{CR}}\right)$$

[3 marks]
1 mark for identifying the differential equation.
1 mark for partially solving the differential equation.
1 mark for solving the differential equation.

b) Substituting $t = 0.004$ s, $V = 1.5$ V, $C = 1 \times 10^{-3}$ F and $R = 1000\ \Omega$ into the equation from 17a) gives:
$$Q=1.5\times1\times10^{-3}\left(1-e^{\frac{-0.004}{1\times10^{-3}\times1000}}\right)$$
$$=2.2\times10^{-5}\text{ C}$$

[2 marks]
1 mark for substituting in the correct values.
1 mark for calculating Q.
Note: Consequential on answer to ***Question 17a)****.*

c) **Method 1 (using 17a):**

$$\frac{dQ}{dt}=\frac{VC-Q}{CR}$$

$$=\frac{1.5\times 1\times 10^{-3}-5\times 10^{-5}}{1\times 10^{-2}\times 1000}$$

$$=0.00145\text{ C s}^{-1}\text{ (or 1.45 mA)}$$

Method 2 (reorganising the initial equation):

$$V-R\frac{dQ}{dt}-\frac{Q}{C}=0$$

$$V-R\frac{dQ}{dt}=\frac{Q}{C}$$

$$R\frac{dQ}{dt}=V-\frac{Q}{C}$$

$$R\frac{dQ}{dt}=\frac{VC}{C}-\frac{Q}{C}$$

$$\frac{dQ}{dt}=\frac{VC-Q}{CR}$$

$$=\frac{1.5\times 1\times 10^{-3}-5\times 10^{-5}}{1\times 10^{-2}\times 1000}$$

$$=0.00145\text{ C s}^{-1}\text{ (or 1.45 mA)}$$

[2 marks]
1 mark for substituting in the correct values.
1 mark for calculating $\frac{dQ}{dt}$.

QUESTION 18 (7 marks)

a) $\mu=3.01$ g

$$\sigma=0.17\times\sqrt{50}$$
$$\approx 1.202$$

[1 mark]
1 mark for calculating μ and σ.
Note: The mean of the population and the mean of the sample are assumed to be identical; therefore, there is no calculation required.

b) $\bar{\bar{X}} = \frac{157}{50}$

$= 3.14 \text{ g}$

$\text{CI} = 3.14 \pm 1.96 \times \frac{1.202}{\sqrt{50}}$

$= 3.14 \pm 0.33$

$= (2.81, 3.47)$

[3 marks]

1 mark for calculating $\bar{\bar{X}}$.

1 mark for selecting the correct formula for the confidence interval and using the correct values.

1 mark for calculating the 95% confidence interval.

*Note: Consequential on answer to **Question 18a)**.*

c) The mean mass of the sugar sachets in a box is the mean mass of a single sachet multiplied by the total number of sachets in a box; thus, 3.01 g × 50 = 150.5 g.

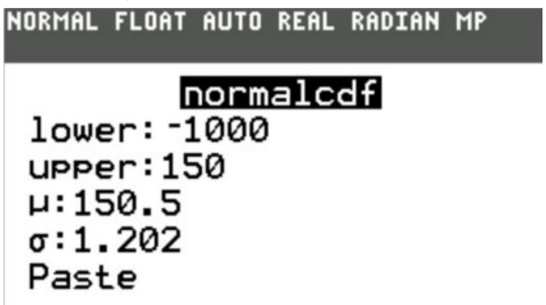
Standard deviation for the box of 50 sachets:

$\sigma = \rho \times \sqrt{50}$

$= 0.17 \times \sqrt{50}$

≈ 1.202

The range for the purposes of calculation using the CDF function on a graphics calculator is –1000 (effectively –∞) to 150.



Therefore, using the CDF function of a graphics calculator, normalCDF (–1000, 150, 150.5, 1.202) gives 0.3387 or 33.87%.

[3 marks]

1 mark for calculating the standard deviation.

1 mark for selecting the correct boundary values to use with the graphics calculator.

Note: This may be implied by the final answer.

1 mark for determining the probability.