Neap

Trial Examination 2022

Suggested Solutions

QCE Specialist Mathematics Units 1&2

Paper 2 – Technology-active

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.			●	
2.			●	
3.	●			
4.				●
5.		●		
6.				●
7.	●			
8.			●	
9.	●			
10.		●		

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QUESTION 1 C

$$\overrightarrow{CB} + 2\overrightarrow{BA} = \left(\overrightarrow{CB} + \overrightarrow{BA}\right) + \overrightarrow{BA}$$
$$= \overrightarrow{CA} + \overrightarrow{BA}$$

QUESTION 2 C

C is correct, and **A** and **B** are incorrect. If a quadrilateral has four sides of the same length, it is a rhombus. If a quadrilateral is a rhombus, then its four sides have the same length. Hence, it can be seen that having four sides of the same length is equivalent to being a rhombus (for a quadrilateral).

$Q \to S$ and $S \to Q$

$\therefore Q \leftrightarrow S$

D is incorrect. The comparison relates to the truth of a statement, not a numerical value.

QUESTION 3 A

When ***a*** is parallel to ***b***, the value of x must be:

$$k\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} x \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} k \\ -3k \end{pmatrix} = \begin{pmatrix} x \\ 1 \end{pmatrix}$$

$$\therefore -3k = 1$$

$$k = -\frac{1}{3}$$

As $k = x$, $x = -\frac{1}{3}$.

When ***b*** is perpendicular to ***c***, the value of y must be:

$$\boldsymbol{b} \cdot \boldsymbol{c} = 0$$

$$\begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ y \end{pmatrix} = 0$$

$$-\frac{2}{3} + y = 0$$

$$y = \frac{2}{3}$$

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QUESTION 4 D

D is correct. By definition, the contrapositive of $\mathrm{P} \rightarrow \mathrm{Q}$ is $\mathrm{Q}' \rightarrow \mathrm{P}'$.

A is incorrect. This option expresses the original implication $\mathrm{P} \rightarrow \mathrm{Q}$. It may be selected if the term contrapositive is misunderstood.

B is incorrect. This option expresses the statement $\mathrm{P}' \rightarrow \mathrm{Q}'$. It may be selected by students who do not understand that contrapositive negates the conditions and alters the implication.

C is incorrect. This option may be selected by students who believe that the negation of P and Q is not P and not Q; that is, neither P nor Q, rather than the correct interpretation of either P′ or Q′.

QUESTION 5 B

As BCE is a straight line, ∠DCE + ∠BCD = 180°. As ABCD is a cyclic quadrilateral, ∠DAB + ∠BCD = 180°. Therefore, $\angle\mathrm{DCE} = \angle\mathrm{DAB}$. Hence, the conclusion can be fully justified using the straight line BCE, as angles on a straight line are supplementary, and the fact that ABCD is a cyclic quadrilateral, as its opposite angles are known to be supplementary.

QUESTION 6 D

Point D shows the complex number 1 – *i*.

$$|1-i| = \sqrt{2}$$

$$\therefore 1 < |z| < 2$$

$$|Arg(1-i)| = \frac{\pi}{4} < \frac{\pi}{2}$$

Therefore, point D shows the position of a compliant complex number.

QUESTION 7 A

$$\begin{aligned} z &= \frac{2\sqrt{3} \pm \sqrt{12-24}}{6} \\ &= \frac{\sqrt{3}}{3} \pm \frac{2\sqrt{-3}}{6} \\ &= \frac{\sqrt{3}}{3} \pm \frac{\sqrt{3}i}{3} \end{aligned}$$

$$\therefore P(z) = A\left(z - \left(\frac{\sqrt{3}}{3} + \frac{\sqrt{3}i}{3}\right)\right)\left(z - \left(\frac{\sqrt{3}}{3} - \frac{\sqrt{3}i}{3}\right)\right)$$

The leading coefficient in the quadratic is 3. Hence, $P(z) = 3\left(z - \left(\frac{\sqrt{3}}{3} + \frac{\sqrt{3}i}{3}\right)\right)\left(z - \left(\frac{\sqrt{3}}{3} - \frac{\sqrt{3}i}{3}\right)\right)$.

QUESTION 8 C

$$\text{amplitude} = |-1|$$
$$= 1$$

$$\text{period} = \frac{2\pi}{\pi}$$
$$= 2$$

For equations with the form $f(x) = D + A\sin\left(B\left(x + C\right)\right)$:

- the parameter D controls the equilibrium level
- the parameter A controls the amplitude, with the amplitude given as $|A|$
- the parameter B controls the period, with the period given as $\frac{2\pi}{B}$
- the parameter C controls the phase shift.

QUESTION 9 A

$$\mathbf{T} = \begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$= 0.5\begin{bmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{bmatrix}$$

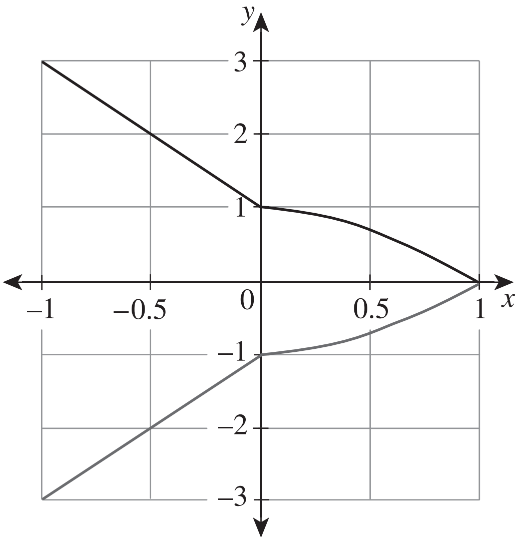
QUESTION 10 B

As $v = w \times 2 \times -i$, two transformations are performed upon w to achieve v. When w is multiplied by 2, the magnitude of w is doubled. When $2w$ is multiplied by $-i$, $2w$ is rotated in the negative direction (clockwise) through $\frac{\pi}{2}$.

SECTION 2

QUESTION 11 (3 marks)

a)



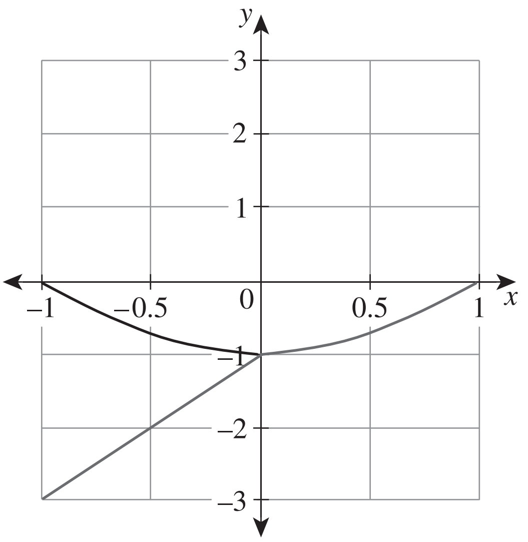
[2 marks]

1 mark for sketching $|f(x)|$ over the domain $-1 < x \le 0$ (straight line).

1 mark for sketching $|f(x)|$ over the domain $0 < x < 1$ (concave down curve).

Note: The grey line represents the graph of f(x) shown in the question.

b)



[1 mark]

1 mark for sketching $f(|x|)$ over the domain $-1 < x < 0$.

Note: The grey line represents the graph of f(x) shown in the question.

QUESTION 12 (6 marks)

a) $\det(\mathbf{M}) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$= 1$

$\det(\mathbf{N}) = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$= -1$

$\det(\mathbf{M}+\mathbf{N}) = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

$= 0$

$$\begin{aligned} \text{LHS} &= \det(\mathbf{M}+\mathbf{N}) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \det(\mathbf{M}) + \det(\mathbf{N}) \\ &= 1 + 1 \\ &= 0 \end{aligned}$$

Therefore, the assertion is true for matrices **M** and **N**.

[3 marks]
*1 mark for providing both det(**M**) = 1 and det(**N**) = –1.*
*1 mark for finding **M** + **N** and providing det(**M** + **N**) = 0.*
*1 mark for stating that the assertion is true for **M** and **N**.*

b) **Method 1:**

Consider allowing $\mathbf{M} = \mathbf{N} = \mathbf{I}_2$.

$$\begin{aligned} \text{LHS} &= \det(\mathbf{M}+\mathbf{N}) \\ &= \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \det(\mathbf{M}) + \det(\mathbf{N}) \\ &= 1 + -1 \\ &= 2 \end{aligned}$$

LHS $\neq$ RHS

Therefore, the existence of this counterexample means that the assertion is not true $\forall \mathbf{M}, \mathbf{N}$.

[3 marks]
*1 mark for providing valid 2 × 2 matrices representing **M** and **N** to act as a counterexample.*
*1 mark for providing the correct determinant calculations for det(**M**), det(**N**) and det(**M** + **N**).*
1 mark for stating that the assertion is not true.

Method 2:

Let $\underset{\sim}{\boldsymbol{M}} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ and $\underset{\sim}{\boldsymbol{N}} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}$ (all elements $\in R$).

$\det(\underset{\sim}{M}) = m_{11}m_{22} - m_{12}m_{21}$

$\det(\underset{\sim}{N}) = n_{11}n_{22} - n_{21}n_{12}$

$\underset{\sim}{M} + \underset{\sim}{N} = \begin{bmatrix} m_{11} + n_{11} & m_{12} + n_{12} \\ m_{21} + n_{21} & m_{22} + n_{22} \end{bmatrix}$

$\det(\underset{\sim}{M} + \underset{\sim}{N}) = (m_{11} + n_{11})(m_{22} + n_{22}) - (m_{12} + n_{12}) \times (m_{21} + n_{21})$

$\text{RHS } \det(\underset{\sim}{M}) + \det(\underset{\sim}{N}) = m_{11}m_{22} + n_{11}n_{22} - (m_{12}m_{21} + n_{21}n_{12})$

$$\begin{aligned}\text{LHS } \det(\underset{\sim}{M} + \underset{\sim}{N}) &= m_{11}m_{22} + n_{11}n_{22} + n_{11}m_{22} + m_{11}n_{22} - (m_{12}m_{21} + n_{12}n_{21} + m_{12}n_{21} + n_{12}m_{21}) \\ &= \det(\underset{\sim}{M}) + \det(\underset{\sim}{N}) + n_{11}m_{22} + m_{11}n_{22} - (m_{12}n_{21} + n_{12}m_{21}) \\ &= \det(\underset{\sim}{M}) + \det(\underset{\sim}{N}) + \Delta \qquad (\Delta \neq 0)\end{aligned}$$

Hence, RHS $\neq$ LHS in general. Therefore, the assertion is not true for all 2 × 2 matrices.

[3 marks]

1 mark for forming $\underset{\sim}{\boldsymbol{M}}$ and $\underset{\sim}{\boldsymbol{N}}$ in terms of their elements.

1 mark for providing the correct expression for $\det(\underset{\sim}{M} + \underset{\sim}{N})$.

1 mark for stating that RHS $\neq$ LHS and determining that the assertion is not true.

QUESTION 13 (6 marks)

a) **Method 1:**

As there is no arrangement of the students within the cabins, the allocation involves selecting three students from the initial nine students for cabin 1, then selecting three students from the remaining six for cabin 2, then selecting the final three students for cabin 3. There is only one way to achieve this final selection, ${}^{3}C_{3} = 1$.

$$\begin{aligned} N &= {}^{9}C_{3} \times {}^{6}C_{3} \times {}^{3}C_{3} \\ &= 1680 \end{aligned}$$

Method 2:

If a position in an arrangement represents a student (for example, position 1 is Ann, position 2 is Beth and so on), the problem involves the arrangement of the cabin numbers 1, 1, 1, 2, 2, 2, 3, 3, 3 into 9 positions. As there are 3 × 1, 3 × 2 and 3 × 3, there are three indistinguishable items that each present 3 times. Hence:

$$\begin{aligned} N &= \frac{{}^{9}P_{9}}{{}^{3}P_{3}\ {}^{3}P_{3}\ {}^{3}P_{3}} \\ &= 1680 \end{aligned}$$

[2 marks]

1 mark for providing valid working that involves combinations, permutations or factorials.

1 mark for providing the correct solution.

b) **Method 1:**

There are nine students in total, and three are allocated to each cabin.

Consider a single student. Two students from the other eight will be allocated to this student's cabin.

$$P(\text{Dee and Heidi in the same cabin}) = \frac{2}{8} = \frac{1}{4} = 0.25$$

Method 2:

Dee and Heidi can be placed into a single 'decision block'. Hence, there are now eight entities to be allocated to the cabins. The decision process of performing the allocation is then:

Step 1: Decide to which cabin (1, 2 or 3) the decision block of Dee and Heidi is to be allocated.

Step 2: Decide which of the other seven students are allocated to Dee and Heidi's cabin.

Step 3: Choose three students from the remaining six to be allocated together.

$$N_{\text{restricted}} = {}_3C_1 \times {}_7C_1 \times {}_6C_3 = 420$$

$$P(\text{Dee and Heidi in the same cabin}) = \frac{420}{1680} = \frac{1}{4} = 0.25$$

[2 marks]
1 mark for providing valid working.
1 mark for providing the correct solution.

c) $P(\text{Dee and Heidi in the same cabin}) = 0.25$

$\therefore P(\text{Dee and Heidi in different cabins}) = 0.75$

$$N' = 0.75 \times 1680 = 1260$$

[2 marks]
1 mark for providing valid working.
1 mark for providing the correct solution.
Note: Consequential on answer to ***Question 13b)****.*

QUESTION 14 (5 marks)

Defining $\angle\text{BOC}$ as 2θ:

ΔOBC is isosceles (OB and OC radii).

$$\begin{aligned}\angle\text{OBC} &= \angle\text{OCB}\\ &= \frac{180-2\theta}{2}\\ &= 90-\theta\end{aligned}$$

Defining $\angle\text{OBA}$ as α:

ΔOBA is isosceles (OB and OA radii).

$\angle\text{BOA} = 180 - 2\alpha$

Defining $\angle\text{OCA}$ as β:

ΔOBA is isosceles (OC and OA radii).

$\angle\text{COA} = 180 - 2\beta$

As $\angle\text{BOC} + \angle\text{BOA} + \angle\text{COA} = 360°$, $\theta = \alpha + \beta$.

As $\angle\text{BAC} = \angle\text{OAB} + \angle\text{OAC}$, $\angle\text{BAC} = \alpha + \beta = \theta$.

Hence, $\angle\text{BOA} = 2\angle\text{BAC}$.

[5 marks]

1 mark for establishing that $\angle OBC = \angle OCB$ using radii.

1 mark for considering at least one of ΔOBA and ΔOBC.

1 mark for using the logic of full revolution at O.

1 mark for reaching $\angle BAC = \angle OAB + \angle OAC$.

1 mark for showing sequential working and correct notation throughout. Note: This mark is to be awarded only if the proof is substantially correct.

QUESTION 15 (9 marks)

a) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

$|\overrightarrow{BC}| = \sqrt{(-3)^2 + (-2)^2}$

$= \sqrt{13}$

≈ 3.606

$\tan(\theta) = \dfrac{-3}{-2}$

$\theta \approx 236.3°$ (4.12 radians)

Using $-180° \leq \theta \leq 180°$:

$\theta = 236.3° - 360°$

$= -123.7°$ (≈ -2.16 radians)

$\overrightarrow{BC} = (\sqrt{13}, 236.3°)$

[3 marks]

1 mark for finding $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$.

1 mark for providing the correct magnitude. Note: Accept responses in exact or decimal form.

1 mark for providing the correct angle.

Note: Accept responses in degrees or radians and that are positive or negative.

b) $\hat{\boldsymbol{v}} = -\dfrac{1}{\sqrt{13}}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\approx \begin{pmatrix} -0.555 \\ -0.832 \end{pmatrix}$

[2 marks]

1 mark for using $\hat{\boldsymbol{r}} = \dfrac{1}{|r|}r$.

1 mark for providing the correct solution in component form.

*Note: Accept responses in exact or decimal form. Consequential on answer to **Question 15a**).*

c) $\begin{bmatrix} 0 & -2 \\ a & 0 \end{bmatrix}\begin{bmatrix} 2 & 4 & 2 \\ 5 & 5 & 2 \end{bmatrix} = \begin{bmatrix} -10 & -10 & -4 \\ 2a & 4a & 2a \end{bmatrix}$

$\therefore A' = (-10, 2a)$, $B' = (-10, 4a)$ and $C' = (-4, 2a)$

[2 marks]

1 mark for attempting an appropriate matrix multiplication.

1 mark for providing the correct solution.

d) $\text{area } \Delta\text{ABC} = 0.5(3 \times 2)$

$= 3$

$\text{area } \Delta\text{A}'\text{B}'\text{C}' = \text{area } \Delta\text{ABC} \times \det(\mathbf{T})$

$\therefore \det(\mathbf{T}) = 3$

$\det(\mathbf{T}) = 2a$

$3 = 2a$

$a = 1.5$

[2 marks]
1 mark for finding that the area of $\Delta ABC = 3$.
1 mark for providing the correct solution.
Note: Accept follow-through errors.

QUESTION 16 (6 marks)

a) Using a graphics calculator:

$$\det\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0.5 & 2 \\ 1 & 1 & -2 \end{bmatrix} = -3$$

[1 mark]
1 mark for providing the correct solution.

b) Using a graphics calculator:

$$6\mathbf{A}^{-1} = 6\begin{bmatrix} 1 & \frac{-4}{3} & \frac{-4}{3} \\ 0 & \frac{2}{3} & \frac{2}{3} \\ \frac{1}{2} & \frac{-1}{3} & \frac{-5}{6} \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -8 & -8 \\ 0 & 4 & 4 \\ 3 & -2 & -5 \end{bmatrix}$$

[2 marks]
1 mark for stating $\mathbf{A}^{-1}$. Note: This may be implied by subsequent working.
1 mark for stating $6\mathbf{A}^{-1}$.

c)
$$\mathbf{AX}+\mathbf{X}=\mathbf{B}$$
$$(\mathbf{A}+\mathbf{I})\mathbf{X}=\mathbf{B}$$
$$\mathbf{X}=(\mathbf{A}+\mathbf{I})^{-1}\mathbf{B}$$

$$(\mathbf{A}+\mathbf{I})^{-1}=\begin{bmatrix}2 & 2 & 0\\ -1 & 1.5 & 2\\ 1 & 1 & -1\end{bmatrix}^{-1}$$
$$=0.1\begin{bmatrix}3 & 2 & 4\\ 6 & 1 & -2\\ 4 & 2 & -1\end{bmatrix}$$

$$\mathbf{X}=(\mathbf{A}+\mathbf{I})^{-1}\mathbf{B}$$
$$=\begin{bmatrix}3 & 2 & 4\\ 6 & 1 & -2\\ 4 & 2 & -1\end{bmatrix}$$

[3 marks]
1 mark for factorising $\boldsymbol{AX}+\boldsymbol{X}$ *to* $(\boldsymbol{A}+\boldsymbol{I})\boldsymbol{X}$.
1 mark for providing $\boldsymbol{X}=(\boldsymbol{A}+\boldsymbol{I})^{-1}\boldsymbol{B}$.
1 mark for showing $\boldsymbol{X}=\begin{bmatrix}3 & 2 & 4\\ 6 & 1 & -2\\ 4 & 2 & -1\end{bmatrix}$.

QUESTION 17 (6 marks)

a) $\overrightarrow{AC}=\boldsymbol{p}+\boldsymbol{q}$

[1 mark]
1 mark for providing the correct solution.

b) $\overrightarrow{BD}=\boldsymbol{q}-\boldsymbol{p}$

[1 mark]
1 mark for providing the correct solution.

c) It is required to prove that $|\overrightarrow{AC}|^2 + |\overrightarrow{BD}|^2 = 2|\boldsymbol{p}|^2 + 2|\boldsymbol{q}|^2$

$$\begin{aligned}\text{LHS} &= |\boldsymbol{q}+\boldsymbol{p}|^2 + |\boldsymbol{q}-\boldsymbol{p}|^2 \\ &= (\boldsymbol{q}+\boldsymbol{p})\cdot(\boldsymbol{q}+\boldsymbol{p}) + (\boldsymbol{q}-\boldsymbol{p})\cdot(\boldsymbol{q}-\boldsymbol{p}) \\ &= \boldsymbol{q}\cdot\boldsymbol{q} + 2\boldsymbol{q}\cdot\boldsymbol{p} + \boldsymbol{p}\cdot\boldsymbol{p} + \boldsymbol{q}\cdot\boldsymbol{q} - 2\boldsymbol{q}\cdot\boldsymbol{p} + \boldsymbol{p}\cdot\boldsymbol{p} \\ &= 2\boldsymbol{p}\cdot\boldsymbol{p} + 2\boldsymbol{q}\cdot\boldsymbol{q} \\ &= 2|\boldsymbol{p}|^2 + 2|\boldsymbol{q}|^2 \\ &= \text{RHS}\end{aligned}$$

Hence, the result is proved.

[4 marks]
1 mark for interpreting what needs to be proved.
1 mark for using a squared modulus that has been transformed to a dot product.
1 mark for cancelling $+2\boldsymbol{q}\cdot\boldsymbol{p}$ and $-2\boldsymbol{q}\cdot\boldsymbol{p}$.
1 mark for showing sequential working and using the correct conventions throughout. Note: This mark is only awarded if the response is substantially correct.

QUESTION 18 (7 marks)

a)

$$\begin{aligned}\overrightarrow{AC} &= \begin{pmatrix}-1\\1.5\end{pmatrix} - \begin{pmatrix}-2\\-3\end{pmatrix} \\ &= \begin{pmatrix}1\\4.5\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\overrightarrow{BC} &= \begin{pmatrix}-1\\1.5\end{pmatrix} - \begin{pmatrix}1\\1\end{pmatrix} \\ &= \begin{pmatrix}-2\\0.5\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\overrightarrow{AC}\cdot\overrightarrow{BC} &= \begin{pmatrix}1\\4.5\end{pmatrix}\cdot\begin{pmatrix}-2\\0.5\end{pmatrix} \\ &= -2 + 2.25 \\ &= 0.25\end{aligned}$$

[2 marks]
1 mark for providing at least one of $\overrightarrow{AC} = \begin{pmatrix}1\\4.5\end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix}-2\\0.5\end{pmatrix}$.
1 mark for providing the correct solution.

b) Using the scalar product to show that $\angle\text{ACB}$ is acute:

$\boldsymbol{a}\cdot\boldsymbol{b}=|a||b|\cos\theta$ (using $\boldsymbol{a}\cdot\boldsymbol{b}=0.25$ from part a))

$$\cos\theta=\frac{0.25}{\sqrt{1^2+4.5^2}\sqrt{(-2)+0.5^2}}$$
$$=0.026307$$
$$\theta=88.5°$$
$$\approx 1.54 \text{ radians}$$

OR

As the scalar product is positive, $\angle\text{ACB}$ is acute.

Therefore:

If point C lies on the circumference, $\angle\text{ACB}$ is a right angle. If point C lies inside the circle, $\angle\text{ACB}$ is obtuse. If point C is outside the circle, $\angle\text{ACB}$ is acute. As the angle is acute, point C lies outside the circle.

[2 marks]
1 mark for showing the calculation of the angle $\angle ACB$ as $88.5° = 1.54$ radians OR providing a valid justification.
1 mark for drawing the conclusion that point C is outside the circle.
*Note: Consequential on answer to **Question 18a)**.*

c)
$$m=\overrightarrow{\text{OM}}$$
$$=0.5(a+b)$$
$$=\begin{pmatrix}-0.5\\-1\end{pmatrix}$$
$$\overrightarrow{\text{MC}}=\begin{pmatrix}-1\\1.5\end{pmatrix}-\begin{pmatrix}-0.5\\-1\end{pmatrix}$$
$$=\begin{pmatrix}-0.5\\2.5\end{pmatrix}$$
$$|\overrightarrow{\text{MC}}|=\sqrt{(-0.5)^2+2.5^2}$$
$$\approx 2.55$$

[2 marks]
1 mark for showing $\overrightarrow{OM}$ as $\begin{pmatrix}-0.5\\-1\end{pmatrix}$.
1 mark for finding $|\overrightarrow{MC}|$.
Note: Accept responses in exact or decimal form.

d) The radius of the circle is $0.5|\overrightarrow{\text{AB}}|=2.5$

As $|\overrightarrow{\text{MC}}|>2.5$ from 18c), it follows that point C lies outside the circle with centre M.

In 18b), the same conclusion was drawn by inspecting the sign of the scalar product $\overrightarrow{\text{AC}}\cdot\overrightarrow{\text{BC}}$.

Hence, the responses are consistent and reasonable.

[1 mark]
1 mark for calculating the radius of the circle to be 2.5 and drawing a conclusion regarding consistency and reasonableness.
*Note: Consequential on answer to **Questions 18a)** and **18b)**.*

QUESTION 19 (7 marks)

a) Let f represent the fishing boat and p represent the patrol boat.

The angles supplied in the question are bearings. Converting the bearings into conventional angles gives 110°T = 340° and 040°T = 50°.

$$\begin{aligned} v_{f\ rel\ p} &= v_f - v_p \\ &= 4\left(\cos(340°)\boldsymbol{i} + \sin(340°)\boldsymbol{j}\right) - 8\left(\cos(50°)\boldsymbol{i} + \sin(50°)\boldsymbol{j}\right) \\ &= -1.383\boldsymbol{i} + -7.496\boldsymbol{j} \end{aligned}$$

$$\left|v_{f\ rel\ p}\right| \approx 7.62 \text{ km/h}$$

$$\begin{aligned} \tan(\theta) &= \frac{-7.496}{-1.383} \\ \theta &= 259.54° \quad \text{(conventional)} \end{aligned}$$

$$270 - 259.54 = 10.46°$$

Therefore, the direction is 10.46° west of south or 79.54° south of west.

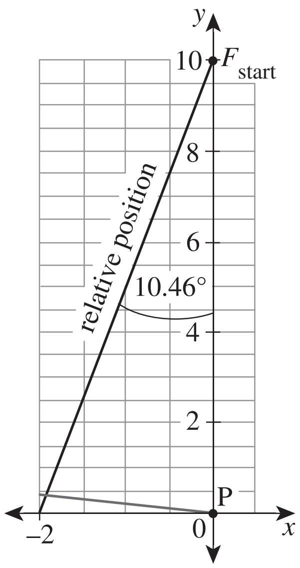
[3 marks]

1 mark for providing $v_{f\ rel\ p} = v_f - v_p$.

1 mark for finding the magnitude of the velocity.

1 mark for finding the direction of the velocity.

b) **Method 1 (relative velocity):**



Consider the relative velocity triangle with a hypotenuse of 10 kilometres and an angle of 10.46°.

The minimum distance between the fishing boat and the patrol boat is seen to be:

$$D_{\text{min}} = 10\sin(10.46°)$$
$$= 1.815 \text{ kilometres}$$

It is stated in the question that, if the fishing boat approaches within two kilometres of the patrol boat, the fishing boat will be seen. Therefore, as 1.815 < 2, it is concluded that the pilot of the patrol boat will see the fishing boat.

[4 marks]

1 mark for sketching a valid diagram that features the use of the initial separation of 10 km OR developing the trigonometric expression in subsequent working.

1 mark for forming a trigonometric equation.

1 mark for providing the expression $D_{min} = 10\sin\theta$ (for some consistent value of θ).

1 mark for providing the correct distance of 1.815 kilometres and concluding that the fishing boat will be seen.

Note: Accept various methods. Consider any valid logic for appropriate credit.

Method 2:

At time 0, the fishing boat is at (0, 10) and the patrol boat is at (0, 0).

At time t, the fishing boat is at $(4\cos(340°)t, 10 + 4\sin(340°)t)$ and the patrol boat is at $(8\cos(50°)t, 8\sin(50°)t)$.

Therefore, the fishing boat is separated from the patrol boat by:

$((4\cos(340°) - 8\cos(50°))t, 10 + 4\sin(340°)t - 8\sin(50°)t)$

Therefore, the distance of separation is:

$$d = \sqrt{(4\cos(340°) - 8\cos(50°))^2 t^2 + (10 + 4t\sin(340°) - 8t\sin(50°)t)^2}$$

By graphing, the minimum is located at $t = 1.29$ hours, $d = 1.815$ km.

As $1.815 < 2$, it is concluded that the pilot of the patrol boat will see the fishing boat.

[4 marks]

1 mark for attempting to formulate at least one position.
1 mark for finding the positions of the two boats at time t.
1 mark for identifying the distance equation.
1 mark for providing the correct distance of 1.815 kilometres and concluding that the fishing boat will be seen.
Note: Accept various methods. Consider any valid logic for appropriate credit.