Neap

Trial Examination 2022

Question and Response Booklet

QCE Specialist Mathematics Units 1&2

Paper 1 – Technology-free

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are **not** permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 9 short response questions

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SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 55 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

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QUESTION 11 (7 marks)

Consider the complex numbers $w = \sqrt{3} - i$ and $q = -2 + 2i$.

a) Write w in polar form. *[2 marks]*

b) Evaluate $\frac{1}{w}$. Give your answer in Cartesian form. *[2 marks]*

c) Evaluate $\frac{q}{w}$. Give your answer in the form $r\,\text{cis}\,\theta$, where θ is an angle in radians. *[3 marks]*

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QUESTION 12 (4 marks)

a) Solve $\frac{{}^{n}C_5}{{}^{n-1}C_4} = 3$ for n. *[2 marks]*

b) Solve $25n = {}^{n}P_2 \times {}^{n-1}P_1$ for n. *[2 marks]*

QUESTION 13 (5 marks)

It is known that $\mathbf{B} = \begin{bmatrix} -2 & 2 & 8 \\ -1 & 2 & 5 \\ -2 & 2 & 6 \end{bmatrix}$, $\mathbf{BA} = \mathbf{AB} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $\mathbf{AX} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

a) State the dimensions of matrix **X**. *[1 mark]*

b) State matrix $\mathbf{A}^{-1}$. *[1 mark]*

c) Determine matrix **X**. *[3 marks]*

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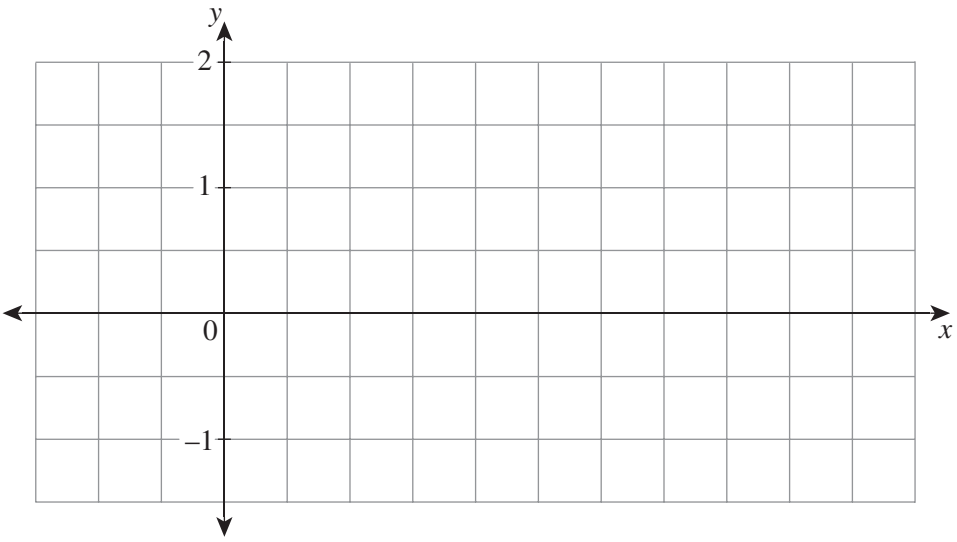
QUESTION 14 (4 marks)

Sketch one full period of the graph of $y = \sin\left(0.5\left(x - \frac{\pi}{2}\right)\right)$. Include numerals on the x-axis and label any axis intercepts.



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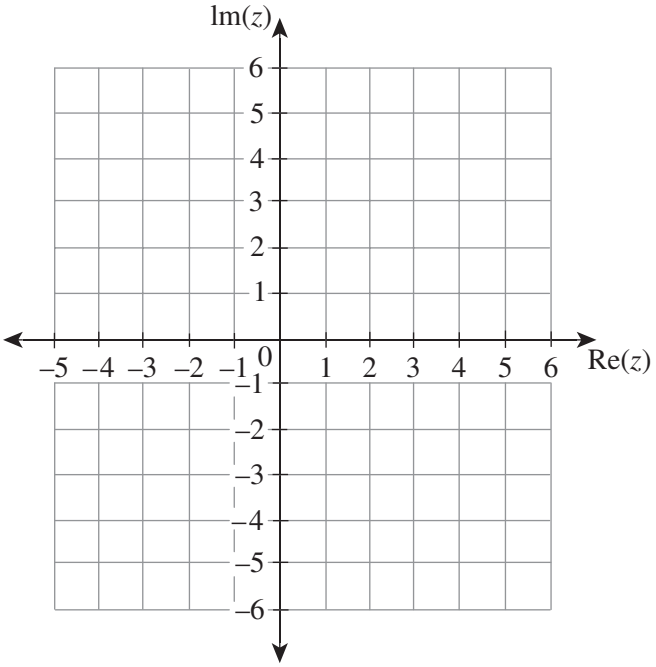
QUESTION 15 (7 marks)

Consider the complex quadratic equation $P(z) = z^2 - 4z + 13$.

a) Find the complex solutions to $P(z) = 0$. *[3 marks]*

b) Plot the complex solutions found in Question 15a) on the Argand diagram below. *[2 marks]*



c) Find the equation of a complex quadratic, $Q(z)$, such that the solutions to $Q(z) = 0$ are the complex conjugate pair $3 + i$ and $3 - i$. *[2 marks]*

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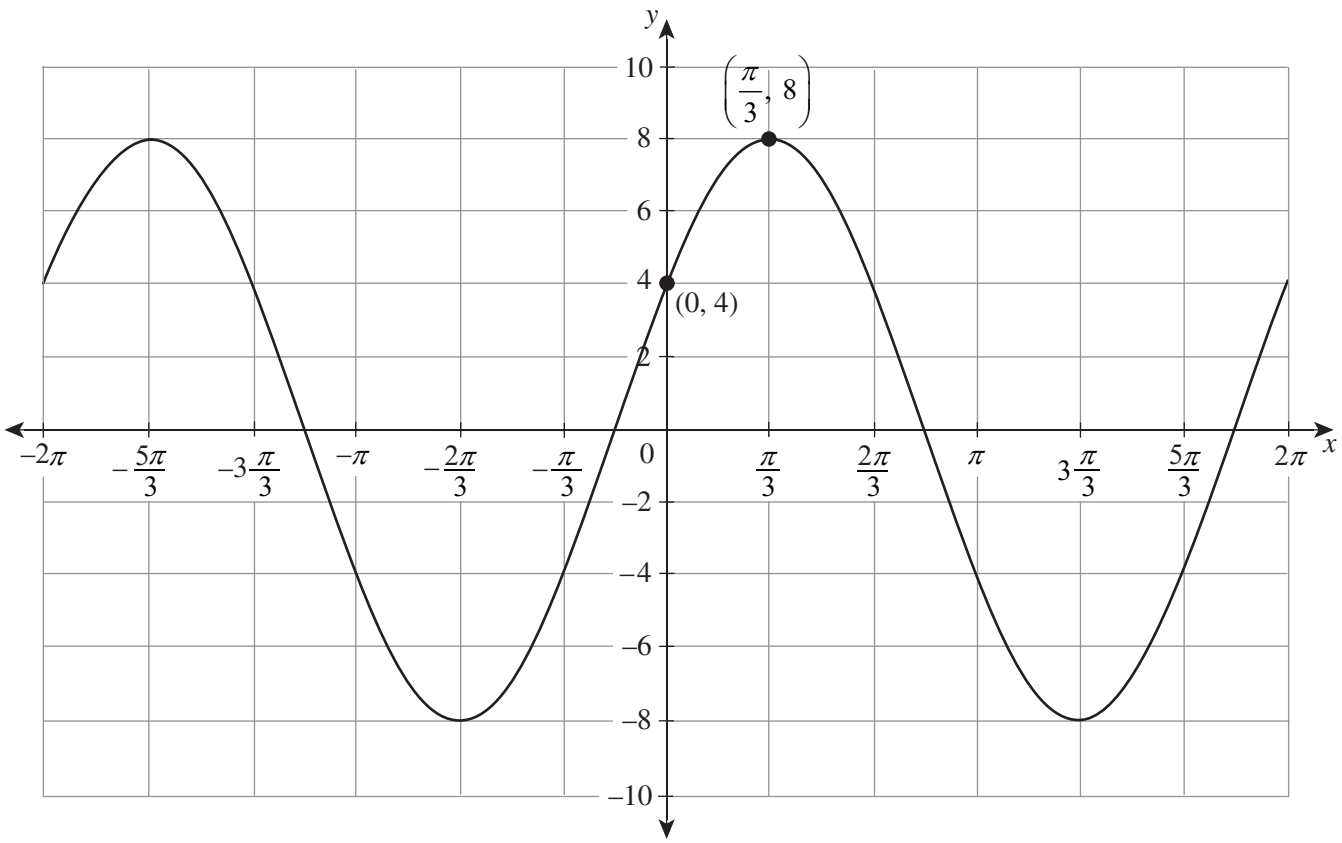
QUESTION 16 (8 marks)

Consider the graph.



a) Given that the equation of the graph has the form $f(x) = A\cos(x - B)$, where $0 < A$ and $0 < B < \frac{\pi}{2}$, determine the values of A and B. *[2 marks]*

b) Given that the equation has the form $g(x) = M\cos x + N\sin x$, determine the values of M and N. *[4 marks]*

c) By evaluating $f\left(\frac{2\pi}{3}\right)$ and $g\left(\frac{2\pi}{3}\right)$ and identifying an appropriate point on the graph, draw a conclusion about the reasonableness of your responses to Questions 16a) and 16b). *[2 marks]*

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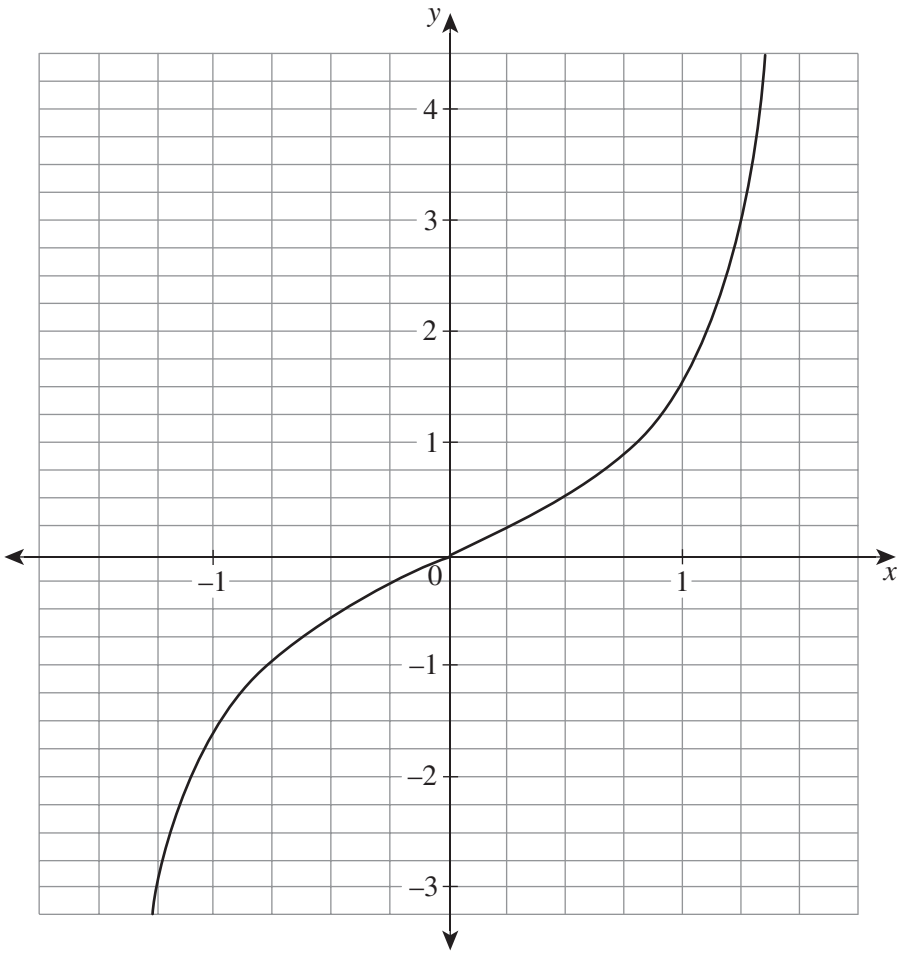
QUESTION 17 (6 marks)

The graph of $y = \tan x$ is shown.

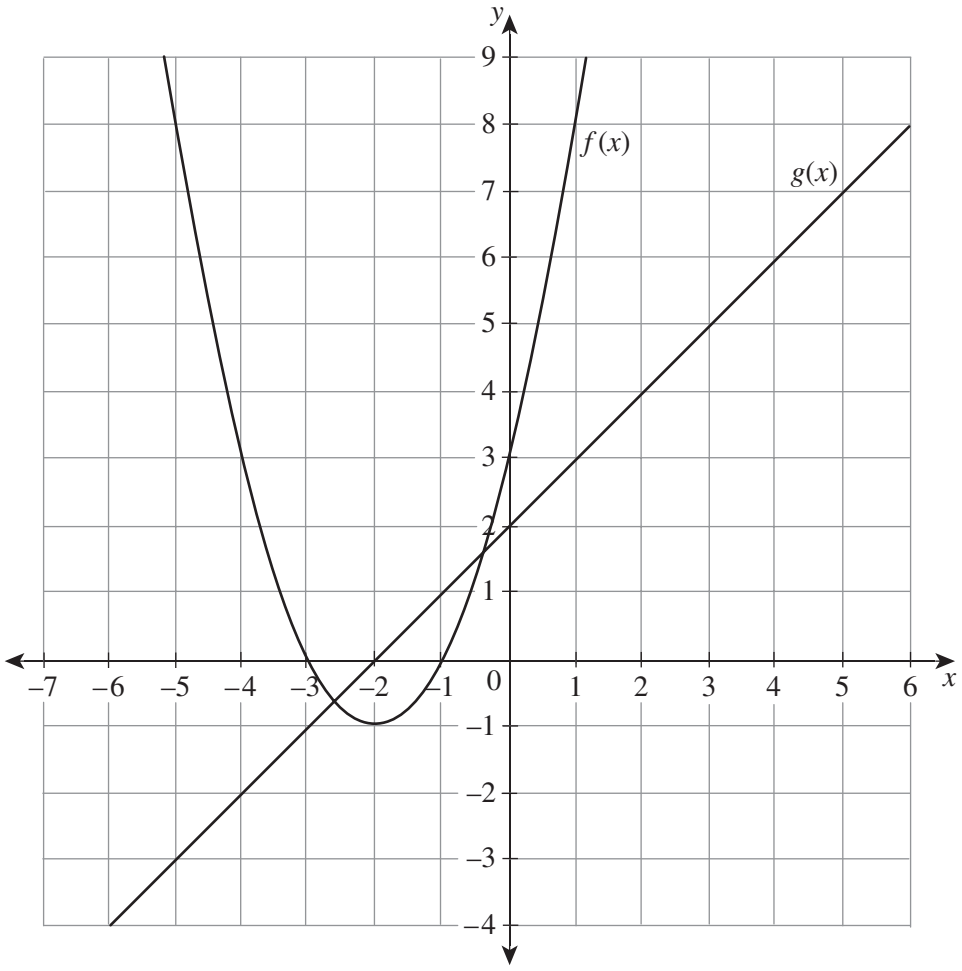


Use the graph to estimate the solutions to $\sec^2(2\theta - 1) - \tan(2\theta - 1) - 7 = 0,\ -0.3 < \theta < 1.3$.

QUESTION 18 (7 marks)

a) Using polynomial division, or otherwise, express the quotient $\frac{x^2+4x+3}{x+2}\,(x \neq -2)$ in the form $ax+b+\frac{c}{x+2},\ a,b,c \in Z$. *[3 marks]*

b) The functions $f(x)=x^2+4x+3$ and $g(x)=x+2$ are shown.



On the axes above, sketch the graph of the quotient obtained in Question 18a). Label any axis intercepts and include the equation(s) for any asymptote(s) that may exist. *[4 marks]*

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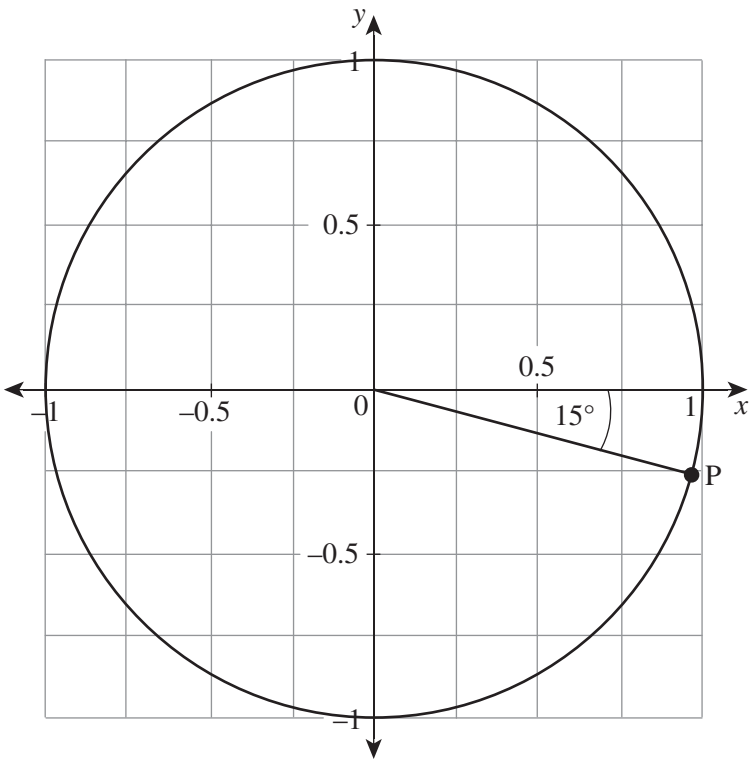
QUESTION 19 (7 marks)

Consider the unit circle shown. The point P is plotted on the circumference of the circle.



a) State the transformation matrix associated with a rotation of 45° about the origin in the clockwise direction. *[2 marks]*

b) Using your solution from Question 19a), or otherwise, state the image, Q, of the point (1, 0) after a rotation of 45° about the origin in the clockwise direction. *[1 mark]*

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c) By identifying a transformation that would map point Q to point P, or otherwise, determine the exact coordinates of point P. *[4 marks]*

END OF PAPER

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Neap

Trial Examination 2022

Formula Booklet

QCE Specialist Mathematics Units 1&2

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Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + c$
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

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Calculus		
chain rule	If $h(x)=f\left(g(x)\right)$ then $h'(x)=f'\left(g(x)\right)g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$
volume of a solid of revolution	about the x-axis	$V=\pi\int_a^b\left[f(x)\right]^2dx$
	about the y-axis	$V=\pi\int_a^b\left[f(y)\right]^2dy$
Simpson's rule	$\int_a^b f(x)dx\approx\frac{w}{3}\left[f\left(x_0\right)+4\left[f\left(x_1\right)+f\left(x_3\right)+\ldots\right]+2\left[f\left(x_2\right)+f\left(x_4\right)+\ldots\right]+f\left(x_n\right)\right]$	
simple harmonic motion	If $\frac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin\left(\omega t+\alpha\right)$ or $x=A\cos\left(\omega t+\beta\right)$	
	$v^2=\omega^2\left(A^2-x^2\right)$	$T=\frac{2\pi}{\omega}$ $\quad f=\frac{1}{T}$
acceleration	$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z=x+yi=r\left(\cos(\theta)+i\sin(\theta)\right)=r\,\mathrm{cis}(\theta)$
modulus	$\left\|z\right\|=r=\sqrt{x^2+y^2}$
argument	$\arg(z)=\theta,\ \tan(\theta)=\frac{y}{x},-\pi<\theta\le\pi$
product	$z_1z_2=r_1r_2\,\mathrm{cis}\left(\theta_1+\theta_2\right)$
quotient	$\frac{z_1}{z_2}=\frac{r_1}{r_2}\mathrm{cis}\left(\theta_1-\theta_2\right)$
De Moivre's theorem	$z^n=r^n\mathrm{cis}\left(n\theta\right)$

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Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
permutation	${}^nP_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \text{cosec}^2(A)$
angle sum and difference identities	$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A - B) - \cos(A + B)\right)$ $\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A - B) + \cos(A + B)\right)$ $\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A + B) + \sin(A - B)\right)$ $\cos(A)\sin(B) = \frac{1}{2}\left(\sin(A + B) - \sin(A - B)\right)$

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Vectors and matrices		
magnitude	$\lvert\boldsymbol{a}\rvert = \left\lvert\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right\rvert = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$	
scalar (dot) product	$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos(\theta)$	
	$\boldsymbol{a} \cdot \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$	
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$	
Cartesian equation of a line	$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\sin(\theta)\hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\cos(\theta)\hat{\boldsymbol{b}} = \left(\boldsymbol{a} \cdot \hat{\boldsymbol{b}}\right)\hat{\boldsymbol{b}}$	
vector equation of a plane	$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\mathbf{A}) = ad - bc$	
multiplicative inverse matrix	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \det(\mathbf{A}) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ m s}^{-2}$

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