Neap

Trial Examination 2022

Suggested Solutions

QCE Specialist Mathematics Units 1&2

Paper 1 – Technology-free

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.	○	○	●	○
2.	○	○	●	○
3.	○	○	○	●
4.	○	○	●	○
5.	○	○	●	○
6.	○	●	○	○
7.	○	●	○	○
8.	○	●	○	○
9.	○	○	○	●
10.	●	○	○	○

QUESTION 1 C

If a number has a recurring decimal, it must be rational. Since the recurring sequence begins in the first decimal position and extends for six digits, it can be expressed as $\frac{285\,714}{999\,999}$, which simplifies to $\frac{2}{7}$.

QUESTION 2 C

The numbers in Pascal's triangle are combinatorial numbers of the form $\binom{\text{row number}}{\text{column number}}$, where the numbering of each row and column begins at 0. The first shaded number is in row (0, 1, **2**) in column (0, 1, **2**), so it can be expressed as $\binom{2}{2}$. The shaded values continue to occupy column 2 and the row increases by 1 each time. Therefore, the shaded numbers can be expressed as $\binom{2}{2}$, $\binom{3}{2}$, $\binom{4}{2}$, …, therefore, $\binom{n+1}{2}$.

QUESTION 3 D

Using the pigeon-hole principle, $80 < \frac{565}{7}$. It is not possible that all trips were within capacity as $80 \times 7 = 560$ and more than 560 passenger trips were made. It is possible that the five extra passengers all went on a single ferry together (exactly one trip), but this cannot be known for certain. Option **D** uses the correct description of 'at least one', which accounts for all potential combinations of the extra passengers.

QUESTION 4 C

bacon (B): $x + y + z + 10 = 120$
lettuce (L): $x + 20 + z + 10 = 110$
tomato (T): $x + y + 2z + 10 = 135$
$B\ \&\ T \rightarrow z = 15$; $B\ \&\ L \rightarrow y = 30$; using $z = 15$ in $L \rightarrow x = 65$

$$\begin{aligned} n(\text{lettuce OR tomato}) &= 30 + 3z + x + y \\ &= 170 \end{aligned}$$

QUESTION 5 C

$$\begin{aligned} |proj| &= 2\sqrt{2} \times \cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right) \\ &= 2\sqrt{2} \times \cos\left(\frac{\pi}{3}\right) \\ &= \sqrt{2} \end{aligned}$$

QUESTION 6 B

$f(|x|) < 0\ \forall x$ such that $-1 < x < 1$

$\therefore \exists a \in R^-$ such that $f(|x|) = a$

QUESTION 7 B

B is correct. The graph shows a reciprocal trigonometric function, where the period of the underlying trigonometric function is π. The asymptotes at $-\frac{\pi}{2}$, 0 and $\frac{\pi}{2}$ imply that the underlying function is $\sin(2x) = 2\sin x \cos x$.

A is incorrect. This option may be reached if it is realised that the base function is a horizontally dilated $\sin(x)$ graph but the dilation factor is mistaken as 0.5 rather than 2.

C is incorrect. This option may be reached if the position of the horizontal dilation and the vertical dilation in the expression of the function is confused.

D is incorrect. This option is the reciprocal of a sine function. However, its period does not match the period shown in the graph.

QUESTION 8 B

To solve **AX** = **B** for **X**, it is necessary to solve $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$. The inverse of $\begin{bmatrix} 2 & a \\ b & 3 \end{bmatrix}$ is $\frac{1}{6-ab}\begin{bmatrix} 3 & -a \\ -b & 2 \end{bmatrix}$.

Therefore, $\begin{bmatrix} x \\ y \end{bmatrix}$ can be found using $\frac{1}{6-ab}\begin{bmatrix} 3 & -a \\ -b & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

QUESTION 9 D

D is correct. The first vector has components 3 and 4. Using 3 : 4 : 5 implies that the complex number has a modulus of 5, $\arg(\pi - \theta)$ (from inspection of the graph). The second vector points in the negative real direction and has a length of 2. Hence, $2\text{cis}(\pi)$. The tip-to-tail nature of vectors implies addition.

A is incorrect. This option would be reached if the operation between the vectors was a subtraction, which reverses the direction of the vector.

B is incorrect. This option may be reached if the conventions for expressing arguments are not fully understood. The diagram shows an angle of size θ, with the rotation being clockwise (that is, in the negative direction).

C is incorrect. This option is logically identical to option **A**; however, the first complex number is written in Cartesian form rather than polar form.

QUESTION 10 A

A is correct. When two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord and its converse.

B and **C** are incorrect. These options show the other possible pairings of products; they do not show the correct chord intervals multiplied together.

D is incorrect. This option is a rearranged version of option **C**.

SECTION 2

QUESTION 11 (7 marks)

a)
$$\begin{aligned} r &= |w| \\ &= \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} \\ &= 2 \end{aligned}$$

$$\tan(\theta) = \frac{-1}{\sqrt{3}}$$

$$\theta = -\frac{\pi}{6}$$

$$\begin{aligned} w &= 2\text{cis}\left(-\frac{\pi}{6}\right) \\ &= 2\text{cis}(-30°) \end{aligned}$$

[2 marks]
1 mark for finding the magnitude of 2.
1 mark for finding the argument of $-\frac{\pi}{6}$ (30°).

b)
$$\begin{aligned} \frac{1}{w} &= \frac{1}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i} \\ &= \frac{\sqrt{3}+i}{\left(\sqrt{3}\right)^2 - (i)^2} \\ &= \frac{\sqrt{3}}{4} + \frac{1}{4}i \end{aligned}$$

[2 marks]
1 mark for providing the conjugate multiplication.
1 mark for providing the correct solution in Cartesian form.

c)
$$\begin{aligned} \frac{q}{w} &= \frac{2\sqrt{2}\text{cis}(135°)}{2\text{cis}(-30°)} \\ &= \sqrt{2}\text{cis}(165°) \\ &= \sqrt{2}\text{cis}\left(\frac{11\pi}{12}\right) \end{aligned}$$

[3 marks]
1 mark for presenting both numbers in the same format (either polar or Cartesian).
1 mark for making a valid attempt to perform the division.
1 mark for providing the correct solution in cis form using radians.
*Note: Consequential on answer to **Question 11a)**.*

QUESTION 12 (4 marks)

a) $$\frac{{}^{n}C_5}{{}^{n-1}C_4} = 3$$

$$\frac{\dfrac{n!}{5!(n-5)!}}{\dfrac{(n-1)!}{4!(n-1-4)!}} = 3$$

$$\frac{\dfrac{n!}{5!}}{\dfrac{(n-1)!}{4!}} = 3$$

$$\frac{n}{5} = 3$$

$$n = 15$$

[2 marks]
1 mark for providing the correct factorial interpretation.
1 mark for providing the correct solution.

b) $$25n = {}^{n}P_2 \times {}^{n-1}P_1$$

$$25n = \frac{n!}{(n-2)!} \times \frac{(n-1)!}{(n-2)!}$$

$$25n = \frac{n(n-1)}{1} \times \frac{(n-1)}{1}$$

$$25n = n(n-1)(n-1) \quad (n \geq 2 \therefore n \neq 0)$$

$$25 = (n-1)(n-1)$$

$$n = 6 \text{ as } n \geq 2$$

[2 marks]
1 mark for achieving n(n –1)(n –1).
1 mark for providing the correct solution.

QUESTION 13 (5 marks)

a) The information provided shows that matrix **A** is 3×3.

Therefore, matrix **X** has dimensions 3×1.

[1 mark]
*1 mark for stating the dimensions of matrix **X**.*

b) As $\mathbf{BA} = \mathbf{AB} = 2I$:

$$\begin{aligned}\mathbf{A}^{-1} &= 0.5\mathbf{B}\\ &= 0.5\begin{bmatrix} -2 & 2 & 8 \\ -1 & 2 & 5 \\ -2 & 2 & 6 \end{bmatrix}\end{aligned}$$

[1 mark]
1 mark for stating matrix $\mathbf{A}^{-1}$.

c) $$\mathbf{AX} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$\begin{aligned}\mathbf{X} &= \mathbf{A}^{-1}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\\ &= 0.5\begin{bmatrix} -2 & 2 & 8 \\ -1 & 2 & 5 \\ -2 & 2 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\\ &= 0.5\begin{bmatrix} 18 \\ 11 \\ 12 \end{bmatrix}\\ &= \begin{bmatrix} 9 \\ 5.5 \\ 6 \end{bmatrix}\end{aligned}$$

[3 marks]
1 mark for stating that inverse pre-multiplies vector.
1 mark for substituting matrix $\mathbf{A}^{-1}$.
1 mark for providing the final answer.
*Note: Consequential on answer to **Question 13b)**.*

QUESTION 14 (4 marks)

$$\text{period} = \frac{2\pi}{0.5}$$
$$= 4\pi$$

amplitude = 1 (read from equation)

Finding the x-intercepts:

$$\sin\left(0.5\left(x - \frac{\pi}{2}\right)\right) = 0$$

Let $\theta = 0.5\left(x - \frac{\pi}{2}\right)$.

Hence, $\sin\theta = 0$.

$$\theta = 0, \pi, 2\pi \ldots = k\pi (k \in z)$$

$$0.5\left(x - \frac{\pi}{2}\right) = k\pi$$

$$x - \frac{\pi}{2} = k(2\pi)$$

$$x = k(2\pi) + \frac{\pi}{2}$$

Using $k = 0, 1, 2$:

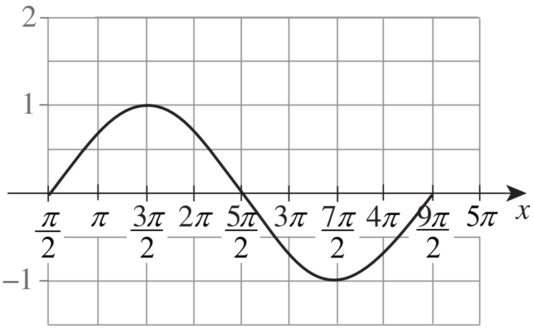
$$x_0 = 0(2\pi) + \frac{\pi}{2} = \frac{\pi}{2}$$

$$x_1 = 1(2\pi) + \frac{\pi}{2} = \frac{5\pi}{2}$$

$$x_2 = 2(2\pi) + \frac{\pi}{2} = \frac{9\pi}{2}$$

Method 1:

Oscillation is presented from $\frac{\pi}{2}$:



[4 marks]

1 mark for finding the correct period.

1 mark for showing the x-intercepts of $\frac{5\pi}{2}$ and $\frac{9\pi}{2}$.

1 mark for sketching a graph with the correct sine shape.

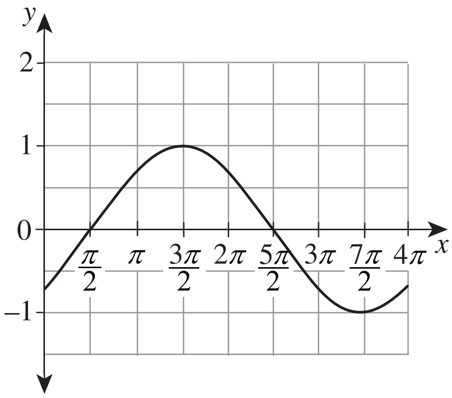
1 mark for showing an amplitude of 1.

Method 2:

Oscillation is presented from 0:

y-intercept:

$$\begin{aligned} y_{\text{int}} &= \sin\left(0.5\left(-\frac{\pi}{2}\right)\right) \\ &= \sin\left(-\frac{\pi}{4}\right) \\ &= -\frac{1}{\sqrt{2}} \end{aligned}$$



[4 marks]

1 mark for finding the correct period.

1 mark for showing the y-intercept of $-\frac{1}{\sqrt{2}}$ *and the x-intercepts of* $\frac{\pi}{2}$ *and* $\frac{5\pi}{2}$.

1 mark for sketching a graph with the correct sine shape.

1 mark for showing an amplitude of 1.

QUESTION 15 (7 marks)

a) **Method 1:**

$$z^2 - 4z + 13 = 0$$
$$(z-2)^2 = -9$$
$$z - 2 = \pm 3i$$

Therefore, the solutions are $2 + 3i$ and $2 - 3i$.

[3 marks]
1 mark for completing the square.
1 mark for reaching $\sqrt{-9} = 3i$.
1 mark for providing the correct solution.

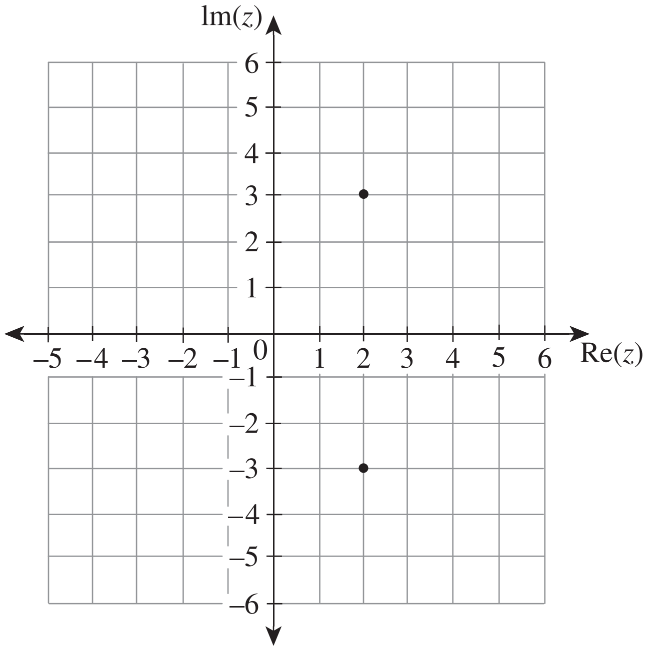
Method 2:

$$z = \frac{4 \pm \sqrt{-36}}{2}$$
$$= \frac{4 \pm 6i}{2}$$

Therefore, the solutions are $2 + 3i$ and $2 - 3i$.

[3 marks]
1 mark for using the quadratic formula.
1 mark for reaching $\sqrt{-36} = 6i$.
1 mark for providing the correct solution.

b)



[2 marks]
1 mark for plotting the first solution.
1 mark for plotting the second solution.
Note: Consequential on answer to ***Question 15a)****.*

c) **Method 1 (quadratic formula):**

$z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (as $z = 3 \pm i$)

$z = \dfrac{3 \pm i}{2(0.5)}$

$b = -3$

$a = 0.5$

$b^2 - 4ac = -1$

$4ac = 10$

$c = 5$

Hence, $Q(z) = A\left(0.5z^2 - 3z + 5\right)$ **OR** $Q(z) = B\left(z^2 - 6z + 10\right)$; $A, B \in R$.

Method 2 (factor theorem):

$$\begin{aligned} Q(z) &= \left(z - (3+i)\right)\left(z - (3-1)\right) \\ &= z^2 - (3+i)z - (3-i)z + (3+i)(3+i) \\ &= z^2 - 6z + \left(3^2 - i^2\right) \\ &= z^2 - 6z + 10 \end{aligned}$$

Method 3 (completing the square):

$z = 3 \pm i \quad \Rightarrow \quad z - 3 = \pm i \quad \Rightarrow \quad (z-3)^2 = (\pm i)^2$

$z^2 - 6z + 9 = -1$

$\therefore Q(z) = z^2 - 6z + 10$

[2 marks]
1 mark for using a valid strategy involving the quadratic formula, factor theorem OR completing the square.
1 mark for providing the correct solution OR a solution that complies with the general form.

QUESTION 16 (8 marks)

a) Reading from the graph, the maximum is 8 and the minimum is –8. Therefore, $A = 8$.

The maximum occurs at $x = \dfrac{\pi}{3}$. Therefore, $B = \dfrac{\pi}{3}$.

[2 marks]
1 mark for finding $A = 8$.
1 mark for finding $B = \dfrac{\pi}{3}$.

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b) Using the supplied points on the graph:

$$M\cos(0)+N\sin(0)=4$$
$$M=4$$

$$4\cos\frac{\pi}{3}+N\sin\frac{\pi}{3}=8$$
$$2+\frac{\sqrt{3}}{2}N=8$$
$$N=6\times\frac{2}{\sqrt{3}}$$
$$=4\sqrt{3}$$

[4 marks]

1 mark for using the evidence provided in the graph (the supplied coordinates or the values of A and B) to form one correct equation.

1 mark for finding M = 4.

1 mark for using the evidence provided in the graph (the supplied coordinates or the values of A and B) to form a second correct equation.

1 mark for finding $N=4\sqrt{3}$.

c) $$f\left(\frac{2\pi}{3}\right)=8\cos\left(\frac{2\pi}{3}-\frac{\pi}{3}\right)$$
$$=8(0.5)$$
$$=4$$

$$g\left(\frac{2\pi}{3}\right)=4\cos\frac{2\pi}{3}+4\sqrt{3}\sin\frac{2\pi}{3}$$
$$=-2+4\sqrt{3}\times\frac{\sqrt{3}}{2}$$
$$=4$$

Reading from the graph, it is observed that when $x=\frac{2\pi}{3}$, $y=4$.

As $f\left(\frac{2\pi}{3}\right)=g\left(\frac{2\pi}{3}\right)=4$, the solutions to 16a) and 16b) are supported by the graph and are consistent. Hence, the responses are reasonable.

[2 marks]

1 mark for evaluating $f\left(\frac{2\pi}{3}\right)$ *and* $g\left(\frac{2\pi}{3}\right)$.

1 mark for drawing a consistent conclusion with reference to the point $\left(\frac{2\pi}{3},4\right)$ *on the graph.*

QUESTION 17 (6 marks)

Using $\tan^2 x + 1 = \sec^2 x$, the equation becomes:

$$\left(\tan(2\theta - 1)\right)^2 - \tan(2\theta - 1) - 6 = 0$$

Using $y = \tan(2\theta - 1)$:

$$y^2 - y - 6 = 0$$
$$(y - 3)(y + 2) = 0$$

$\therefore \tan(2\theta - 1) = 3$ **OR** $\tan(2\theta - 1) = -2$

Reading from the graph, there are points located at approximately (1.25, 3) and (–1.1, –2).

Hence:

$2\theta - 1 \approx 1.25$

$\theta \approx 1.125$

OR

$2\theta - 1 \approx -1.1$

$\theta \approx -0.05$

It is noted that these are the only solutions that will exist in the domain $-0.3 < \theta < 1.3$.

[6 marks]

1 mark for stating $\tan^2 x + 1 = \sec^2 x$.

1 mark for identifying the quadratic.

1 mark for obtaining the solutions of 3 and –2 for the quadratic.

1 mark for identifying the appropriate points of the graph; for example, (1.25, 3) and (–1.1, –2). Note: Accept follow-through errors. This mark may be implied by subsequent working.

1 mark for equating the solutions obtained from the graph to $2\theta - 1$ OR showing evidence of transforming the graph using a horizontal dilation of 0.5 and a phase shift of 0.5 to the right (may be apparent in an effort to redraw the graph).

1 mark for providing the correct solutions.

Note: Accept responses in the ranges $-0.1 \le \theta_1 \le 0.02$ and *$1.10 \le \theta_2 \le 1.13$.*

QUESTION 18 (7 marks)

a) **Method 1 (polynomial division):**

$$\begin{array}{r} x+2-1 \\ x+2\overline{\big)\,x^2+4x+3} \\ -(x^2+2x) \\ \hline 2x+3 \\ -(2x+4) \\ \hline -1 \end{array}$$

$$\therefore x+2-\frac{1}{x+2}$$

Method 2 (recognition):

$$x^2+4x+3=x^2+4x+4-1$$

$$x^2+4x+3=(x+2)^2-1$$

$$\frac{x^2+4x+3}{x+2}=\frac{(x+2)^2}{x+2}-\frac{1}{x+2}$$

$$=x+2-\frac{1}{x+2}$$

Method 3 (coefficient matching):

$$x^2+4x+3=(x+2)(ax+b)+c$$

matching on

x^2: $\quad 1=a$

x^1: $\quad 4=2a+b$

$4=2(1)+b$

$2=b$

x^0: $\quad 3=2b+c$

$3=2(2)+c$

$-1=c$

$$\therefore x+2-\frac{1}{x+2}$$

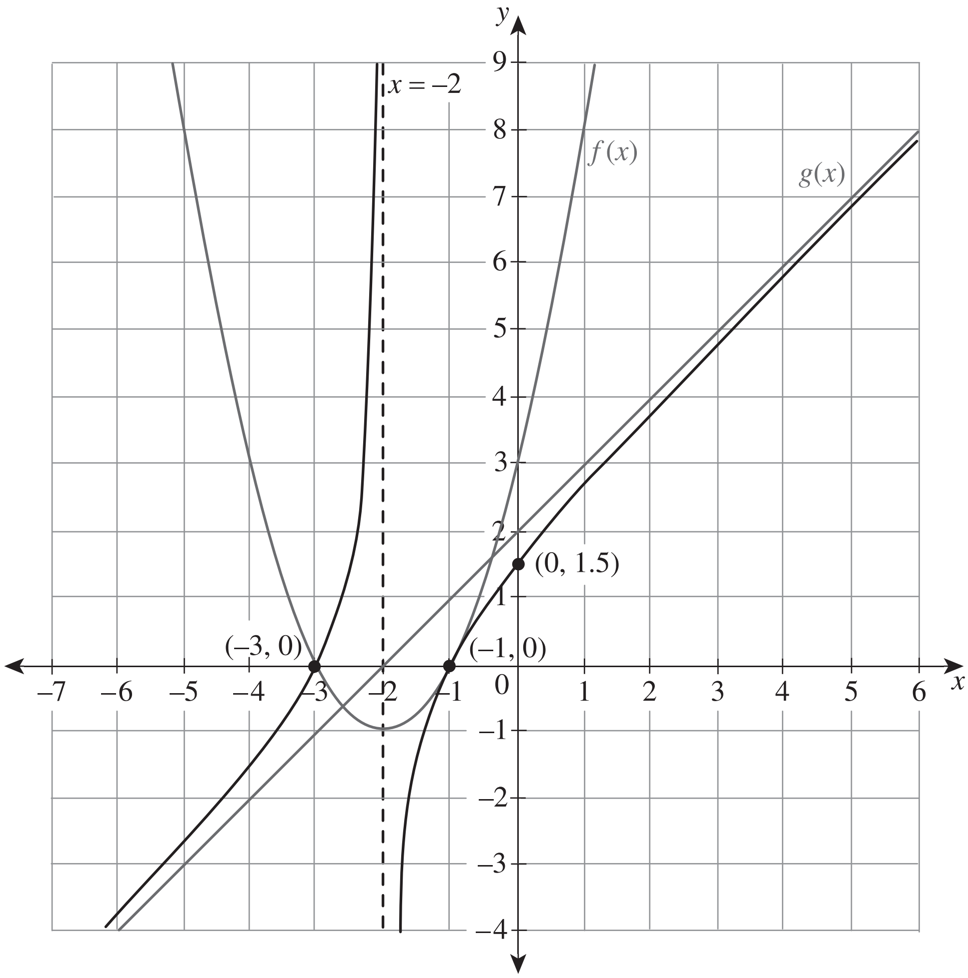
[3 marks]

1 mark for using a valid strategy involving polynomial division, recognition or coefficient matching.

1 mark for achieving a remainder of –1.

1 mark for expressing the quotient in the specified form.

b)



[4 marks]

1 mark for showing the y-intercept of 1.5.

1 mark for showing the x-intercepts of –1 and –3.

1 mark for providing the equation of the vertical asymptote at $x = -2$.

1 mark for showing the rational function approaching the existing straight line $x + 2$ from below as $x \to \infty$ (the oblique asymptote of $y = x + 2$ need not be identified explicitly) AND showing the rational function approaching the existing straight line $x + 2$ from above as $x \to -\infty$.

QUESTION 19 (7 marks)

a) $$\mathbf{T} = \begin{bmatrix} \cos(-45°) & -\sin(-45°) \\ \sin(-45°) & \cos(-45°) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

[2 marks]

1 mark for showing that every element has a magnitude of $\frac{1}{\sqrt{2}}$.

1 mark for providing the correct matrix.

b) **Method 1 (linear transformations):**

$$\mathbf{Q} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$
$$= \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

Method 2 (unit circle ideas):

Q is a point associated with a rotation of –45° from (1, 0).

$\therefore$ Q = (cos(–45°), sin(–45°))

$$= \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

[1 mark]

1 mark for providing the correct solution.

c) **Method 1 (linear transformations):**

A transformation that will map **Q** to **P** is a counterclockwise rotation through 30°.

$$\mathbf{S} = \begin{bmatrix} \cos(30°) & -\sin(30°) \\ \sin(30°) & \cos(30°) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\mathbf{P} = \mathbf{SQ}$$
$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1+\sqrt{3}}{2\sqrt{2}} \\ \frac{1-\sqrt{3}}{2\sqrt{2}} \end{bmatrix}$$

[4 marks]
1 mark for identifying the rotation through 30°.
1 mark for correctly evaluating the matrix.
1 mark for presenting $\boldsymbol{P} = \boldsymbol{SQ}$ *and attempting a matrix multiplication.*
1 mark for providing the correct solution.
Note: Accept two fractions versus common denominator; accept a non-rational denominator.

Method 2 (unit circle reasoning and trigonometric identities):

P is a point associated with a rotation of –15°.

Therefore, the coordinates of P are (cos(–15°), sin(–15°)).

$$\begin{aligned}\cos(-15°) &= \cos(15°)\\ &= \cos(45° - 30°)\end{aligned}$$

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\begin{aligned}\cos(45 - 30) &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\\ &= \frac{\sqrt{3}+1}{2\sqrt{2}}\end{aligned}$$

$$\sin(-15°) = \sin(30° - 45°)$$

$$\sin(A - B) = \sin A \cos B + \cos A \sin B$$

$$\begin{aligned}\sin(A - B) &= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\\ &= \frac{1-\sqrt{3}}{2\sqrt{2}}\end{aligned}$$

[4 marks]

1 mark for identifying the coordinates of P.

1 mark for applying the angle sum or difference to find cos(±15°).

1 mark for applying the angle sum or difference to find sin(±15°).

1 mark for providing the correct solution.

Note: Accept two fractions versus common denominator; accept a non-rational denominator.

