Neap

Trial Examination 2023

Question and Response Booklet

QCE Mathematical Methods Units 3&4

Paper 2 — Technology-active

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (45 marks)

- 10 short response questions

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2023 QCE Mathematical Methods Units 3&4 Written Examination.

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 45 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

The following function is a probability density function for the continuous variable X.

$$f(x)=\begin{cases}\frac{1}{8}x, & 0\le x\le a\\ 0, & \text{otherwise}\end{cases}$$

a) Determine the value of a. *[1 mark]*

b) Determine the expected value of the distribution. *[1 mark]*

c) Determine the standard deviation of the distribution. *[2 marks]*

QUESTION 12 (5 marks)

pH is a measurement of the acidity or alkalinity of a water-based solution. pH values range from 0 to 14.

The function for pH is usually expressed as $pH = -\log_{10}H^+$, where H^+ is the concentration of hydrogen ions measured in moles per litre.

If a solution's pH is 7, the solution is neutral. A pH greater than 7 indicates that a solution is alkaline, while a pH less than 7 indicates that a solution is acidic.

a) Determine the pH of a solution with a hydrogen ion concentration of 0.003 moles per litre. Identify whether the solution is acidic or alkaline. *[2 marks]*

b) A second solution has a pH that is 5 points higher on the pH scale than the solution in Question 12a).

Determine the hydrogen ion concentration of this second solution. *[1 mark]*

c) Multiplying the hydrogen ion concentration of a solution by a factor of k decreases the solution's pH by 3.5.

Determine the value of k. *[2 marks]*

QUESTION 13 (6 marks)

The velocity of a particle is modelled by the function $v(t) = e^{-2t} + t$, where:

- v is the velocity in μm s^{-1}
- $t > 0$
- t is the time in seconds.

a) Determine the acceleration function of the particle. *[1 mark]*

b) Determine the displacement function of the particle if the particle has a displacement of 0 μm at 0 seconds. *[2 marks]*

c) Using mathematical justification, show that the particle is never at rest. *[2 marks]*

d) Determine the value of t when the velocity of the particle reaches its minimum. You are not required to categorise the point as a minimum in your response. *[1 mark]*

QUESTION 14 (7 marks)

In a clinical trial, medication A was administered and absorbed into the bloodstream. Based on the data of multiple test subjects, a model has been developed. The instantaneous rate of change of the mass of medication A circulating in the bloodstream is modelled using the function $M'(t) = 3te^{-0.5t}$, where:

- M is the mass of the medication circulating in the bloodstream in nanograms
- $t > 0$
- t is the time in hours.

a) Determine the maximum instantaneous rate of change of the mass of medication A circulating in the bloodstream and the time when it occurs. *[1 mark]*

b) Determine the duration of time that the instantaneous rate of change of the mass of medication A is greater than 1.5 nanograms per hour. *[2 marks]*

c) Determine the total mass of medication A that circulates in the bloodstream over the first 10 hours. *[2 marks]*

d) Determine the average rate of change of the mass of medication A circulating in the bloodstream over the first 10 hours. *[1 mark]*

e) Total body exposure is the total mass of a medication that has circulated in the bloodstream after one dose is administered.

Comment on the accuracy of the function $M'(t) = 3te^{-0.5t}$ for modelling the total body exposure for a patient who has had one dose of medication A. Support your answer by referring to one graphical feature of the model. *[1 mark]*

QUESTION 15 (3 marks)

The populations of two species that live in a large tidal pool have been modelled.

- A population of crabs has been modelled using $C(t) = -e^{\frac{t-10}{5}} + \gamma$, where $0 < t < 52$ weeks and γ is a constant.
- A population of octopuses has been modelled using $B(t) = 6\ln\left(\frac{t+2}{4}\right) - \alpha t^2 + 21$, where $0 < t < 52$ weeks and α is a constant.

The population of crabs will be 0 at 26.5 weeks, and the population of octopuses will peak exactly 10 weeks before this occurs.

Determine when the two populations will be the same size.

QUESTION 16 (3 marks)

A survey found that 46 out of 150 people suffer from insomnia. A confidence interval for the population proportion needs to be determined with a margin of error between 0.03 and 0.05.

Determine the range of confidence levels that can be achieved with this constraint.

QUESTION 17 (5 marks)

A triangular shade cloth has been put together to cover some small plants in a garden. Two of the triangle's vertices are tied to the top of wooden stakes that have a height of 2 metres. The stakes are perpendicular to the ground. The third vertex of the triangle is fixed to a point on the ground; this point is 6 metres away from one stake and 4.5 metres away from the other. The area of the triangle formed on the ground by the stakes and the point is 13.5 m^2.

Determine the angle of the vertex of the triangular shade cloth that is affixed to the ground. Use diagrams and calculations to support your answer.

QUESTION 18 (4 marks)

Vanessa scored 80% in her English exam and 85% in her History exam. The following information is known.

- The results for both exams are roughly normally distributed.
- The probability of scoring more than 45% in the English exam was 89%.
- The probability of scoring less than 70% in the English exam was 80%.
- The probability of scoring more than 65% in the History exam was 50%.
- The probability of scoring between 45% and 85% in the History exam was 82%.

Determine which subject Vanessa scored comparatively better in.

QUESTION 19 (3 marks)

A manufacturer produces tins of sardines; the tins are sold in boxes of two. The manufacturer runs many production batches each day at its factory.

The manufacturer recently learned that its tins of sardines may be contaminated with trace amounts of toxin A. However, the toxin has not yet been detected in any production batch. To test for contamination, one box of sardines is taken aside for testing at the beginning of each production batch. Both tins from the box are tested for traces of toxin A as well as other contaminants. If either tin is found to be contaminated, the entire production line will be shut down.

For each production batch, the probability that a tin of sardines is contaminated is normally distributed with a mean of 0.25 and a standard deviation of 0.05. The following information is also assumed.

- The **only** possible contaminant is toxin A.
- The contaminant test is 100% accurate.

A production batch is about to begin.

Determine the probability that the production line will have a less than 30% chance of being shut down.

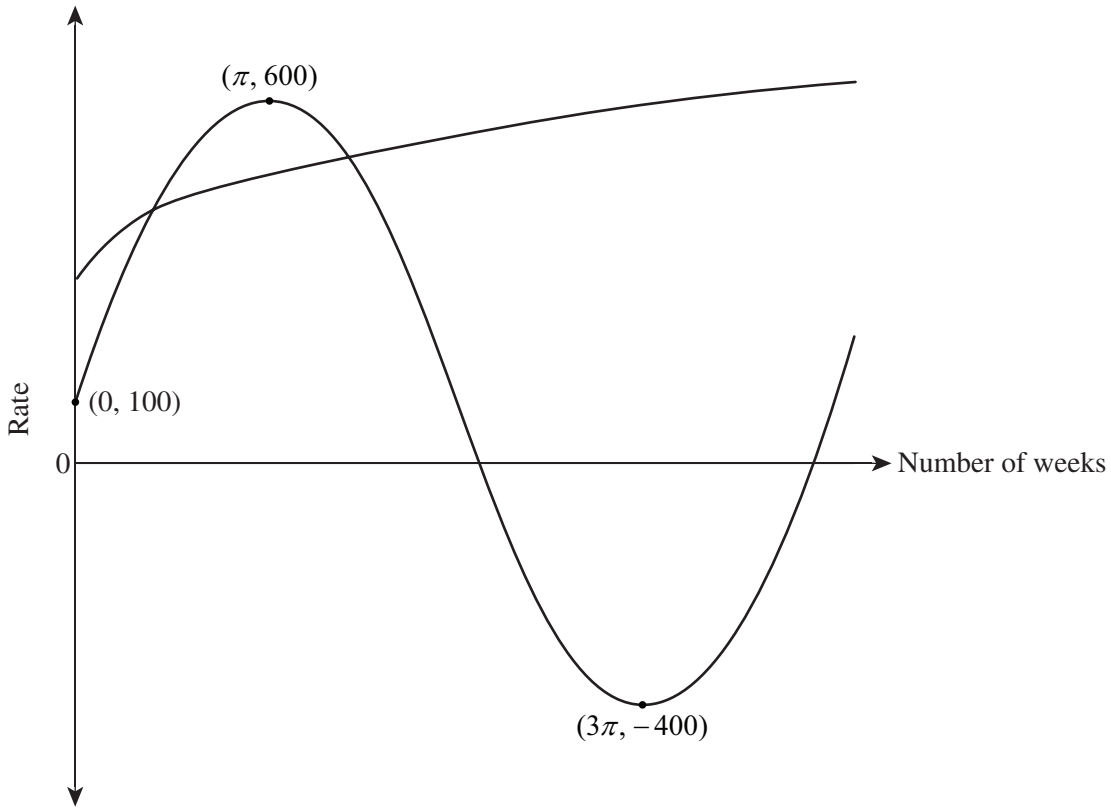
QUESTION 20 (5 marks)

Felix has two channels on a video-sharing website. He discusses politics on one of his channels, and fashion on the other channel. His daily subscriber figures were converted into weekly rates of increase or decrease and were modelled as follows.

- The politics channel subscription rate was modelled using a trigonometric curve.
- The fashion channel subscription rate was modelled using a function in the form $y = 200\log_a(t + b) + 300$, where t represents the number of weeks since the start of the year.

Felix noticed that the two channels had the same subscription rate on days 9 and 31, as shown in the graph.

Rate of number of subscribers for the first 12 weeks of 2023



Felix claimed that he gained more than 7000 subscribers in total over the first 10 weeks of 2023.

Evaluate the reasonableness of his claim.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Write the question number you are responding to.

Neap

Trial Examination 2023

Formula Booklet

QCE Mathematical Methods Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	