Neap

Trial Examination 2023

Question and Response Booklet

QCE Mathematical Methods Units 3&4

Paper 1 — Technology-free

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are not permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (45 marks)

- 9 short response questions

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2023 QCE Mathematical Methods Units 3&4 Written Examination.

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 45 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

Solve the following equations for x.

a) $\ln x + \ln(x + 1) = \ln 20$ *[2 marks]*

b) $e^4 \times \left(\left(e^{x-2}\right)^2 - e^{12}\right) = 0$ *[2 marks]*

QUESTION 12 (3 marks)

Determine the derivative of each of the following with respect to x.

a) $y = \cos(x) \times \ln(x)$ *[1 mark]*

b) $y = \dfrac{\sin(x)}{e^{2x}}$

Simplify your answer. *[2 marks]*

QUESTION 13 (6 marks)

Consider the function $y = 27 - 3x^2$.

a) Write a definite integral that represents the area between the function and the x-axis. *[2 marks]*

b) Determine the area between the function and the x-axis. *[2 marks]*

c) Use the trapezoidal rule to estimate the area between the function and the x-axis.
Use six intervals with equal widths of one unit. *[2 marks]*

QUESTION 14 (5 marks)

At a carnival, a stall vendor runs a game of chance where a player selects three balls from a barrel. After selecting a ball from the barrel, the player then places the ball back in the barrel. There are the same number of red balls and gold balls in the barrel. The player wins a prize if they select at least one gold ball. The more gold balls that a player selects, the better the prize.

a) Determine the expected value for the number of gold balls that would be drawn by a player. *[1 mark]*

b) Determine the standard deviation for the number of gold balls that would be drawn by a player. *[1 mark]*

c) The vendor claims that 9 out of 10 players win a prize by playing the game once.
Use calculations to evaluate the vendor's claim. *[3 marks]*

QUESTION 15 (4 marks)

Every year, school students participate in nationwide competitions for Mathematics and Chemistry. The tests are scored out of 100 marks and students are awarded one of the following achievement levels: high distinction, distinction, credit and participation.

The achievement levels are not determined using the students' test scores. Instead, they are determined based on whether a student's score is in the top 10%, top 30%, top 50% or lowest 50% of all participants.

The following can be assumed.

- The scores for the Mathematics and Chemistry competitions are normally distributed.
- For Mathematics, the mean is 45 and the standard deviation is 15.
- For Chemistry, the mean is 51 and the standard deviation is 20.

a) Aleyna participated in both competitions this year. She scored 70 for Mathematics and 76 for Chemistry.

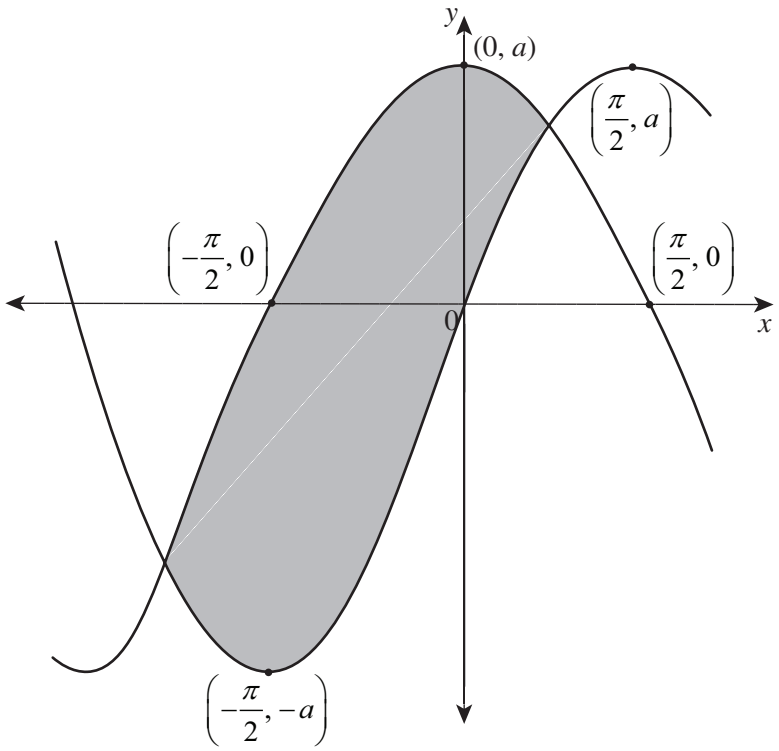
Determine the competition in which Aleyna achieved the comparatively better result. *[3 marks]*

b) To be awarded a distinction, a student's score must be in the top 30%.

Determine the minimum score that a student would need to obtain to be awarded a distinction in the Mathematics competition. Assume that $P(z < 0.5) = 70\%$, where z is the standard normal variable. *[1 mark]*

QUESTION 16 (6 marks)

The graphs of two trigonometric functions are shown below. The shaded area is $6\sqrt{2}$ units^2.

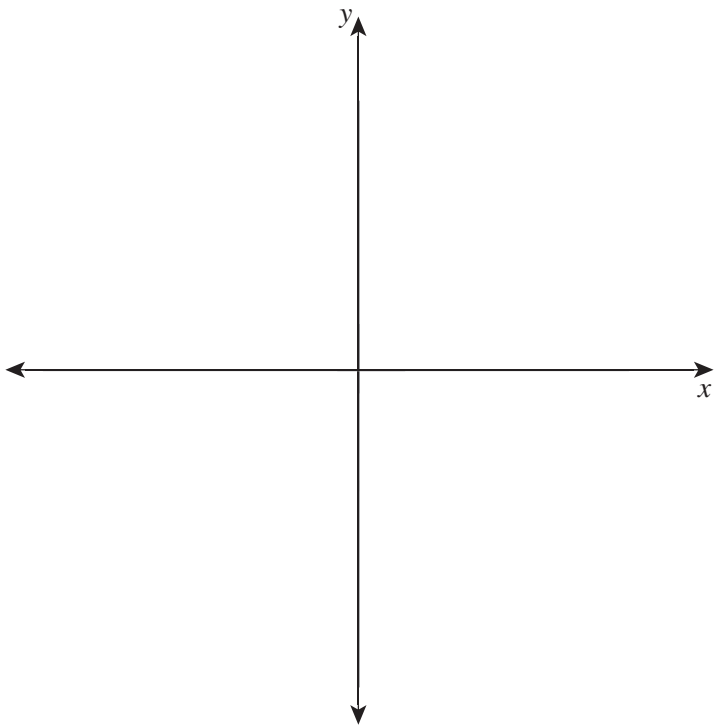


Determine the value of a.

QUESTION 17 (6 marks)

Consider the function $y = \ln\left(x^2 + 2\right)$.

Sketch the graph of the function on the axes below. Determine and label the coordinates of all intercepts, points of inflection and local maxima or minima. Provide justification for the determination of these points and for the categorisation of local maxima or minima using the second derivative test.



QUESTION 18 (6 marks)

Congruent triangles are triangles that have the same shape and size, and isosceles triangles are triangles that have at least two sides that are the same length. An eight-sided polygon (octagon) is to be made using eight congruent triangles. The corresponding vertices meet at the centre of the octagon, and the total area of the octagon is 8 units^2.

Show that the octagon has a minimum perimeter if the triangles are isosceles triangles. In your answer, provide justification that the perimeter is the smallest possible.

QUESTION 19 (5 marks)

Hyperbolic functions are similar to trigonometric functions, except they relate to the hyperbola instead of the unit circle. Hyperbolic functions are commonly expressed using exponential functions; two such hyperbolic functions are as follows.

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

The acceleration of a particle has been modelled using the following function for $t > 0$ seconds.

$$a(t) = \frac{2\sinh(\ln t)}{\ln\left(2\cosh t - e^{-t}\right)} \text{ m s}^{-2}$$

At 1 second, the velocity of the particle is 2 m s^{-1} and its displacement is $\frac{e^{-4}+1}{2}$ m.

Determine the displacement of the particle at e^2 seconds. Simplify your answer by expressing it using one of the hyperbolic functions shown above.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Write the question number you are responding to.

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Formula Booklet

QCE Mathematical Methods Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	