Neap

Trial Examination 2023

Question and Response Booklet

QCE Mathematical Methods Units 1&2

Paper 2 — Technology-active

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (45 marks)

- 10 short response questions

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SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 45 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

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QUESTION 11 (4 marks)

a) Simplify the expression $\frac{4^2 \times 8}{2^5}$ using index laws. *[2 marks]*

b) Consider the following equation.

$$\log_3(2x - 1) = 1.7$$

Make x the subject of the equation and hence, or otherwise, solve for x. Give your answer correct to one decimal place. *[2 marks]*

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QUESTION 12 (4 marks)

a) Solve $2x^2 - 8x + 7 = 0$ using the quadratic formula. *[2 marks]*

b) Solve $x^2 + 8x = 5$ by completing the square. *[2 marks]*

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QUESTION 13 (4 marks)

A tangent is sketched to the curve $y = \frac{1}{2x^2 + 1}$. The equation of the tangent is $y = -\frac{1}{8}x + c$.

Determine all possible values of c.

QUESTION 14 (4 marks)

Mei travels to school on a bus. Some days Mei is late, and some days it is raining. Let L be the event that Mei is late and R be the event that it is raining. It is known that $P(L) = 0.15$, $P(R) = 0.22$ and $P(L \mid R) = 0.45$.

a) Determine $P(L \cap R)$. *[2 marks]*

b) Show that events L and R are not independent. *[2 marks]*

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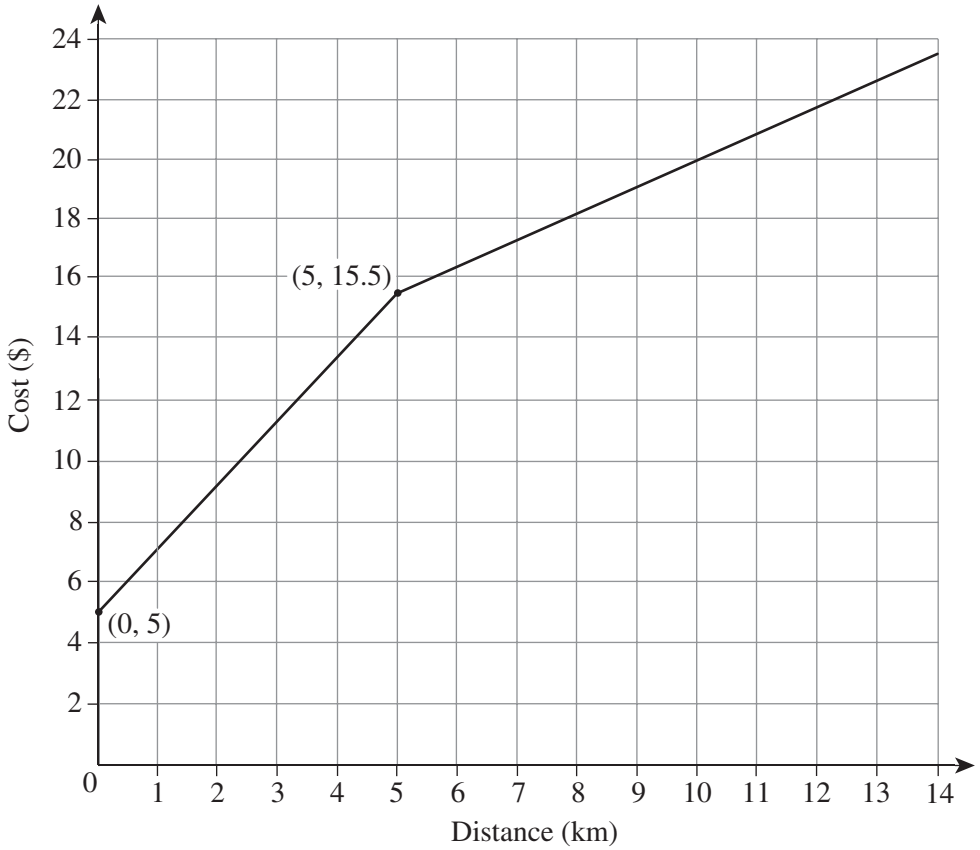
QUESTION 15 (3 marks)

A ride share company charges the following prices based on the distance travelled.



Write a piecewise function to represent the cost in dollars, C, based on the distance travelled in kilometres, x.

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QUESTION 16 (5 marks)

In a particular location, the probability that it will rain tomorrow depends on the weather today, such that:

- if it rained today, there is a 40% chance it will rain tomorrow.
- if it did not rain today, there is a 15% chance that it will rain tomorrow.

Today was a dry day.

a) Sketch a probability tree representing the probability of rain over the next three days. *[2 marks]*

b) Using the probability tree from 16a), or otherwise, determine the following.

i) the probability that it will be dry for the next three days *[1 mark]*

ii) the probability that it will rain on at least one day *[1 mark]*

iii) the probability that it will rain on the third day from today *[1 mark]*

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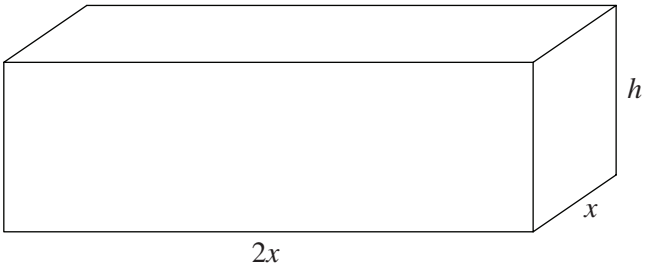
QUESTION 17 (5 marks)

A brick is made such that its width is twice the size of its depth, as shown in the following diagram. The height, h, of the brick varies independently of the base dimensions. All lengths are measured in centimetres.



Not to scale

A company is planning to manufacture this type of brick using exactly 670 cm^3 of material per brick.

a) Show that the surface area of each brick, S, is given by $S = 4x^2 + \frac{2010}{x}$. *[2 marks]*

b) Determine the height, h, of a brick with a minimum surface area. *[3 marks]*

QUESTION 18 (5 marks)

It is known that a geometric sequence is constructed such that $t_8 = 3 \times t_3$.

a) Determine the value of the common ratio, r. *[2 marks]*

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b) It is also known that $t_8 = 3 + t_3$.
Determine the value of S_{10}. *[3 marks]*

QUESTION 19 (5 marks)

Consider the following factorised algebraic expression.

$$(1 + ax)(3+ 2x)^6$$

When the expression is fully expanded, the coefficient of x^2 is –243.

Determine the value of a.

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QUESTION 20 (6 marks)

A lake in a national park contains trout fish that are important for the biodiversity of the lake. At the beginning of 2010, a park ranger estimated that there were 3000 trout in the lake. In 2023, the estimate is approximately 1000 trout. The park rangers have decided to implement one of the following plans to limit the decline of the trout population.

- Plan A: Immediately add 200 trout to the population and add a further 20 trout per year.
- Plan B: Improve the water quality, which in turn will decrease the rate at which the population of trout is declining by 30%.

Determine which plan is better by considering the population forecast for 10 years' time. Discuss the reasonableness of your conclusion.

END OF PAPER

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Neap

Trial Examination 2023

Formula Booklet

QCE Mathematical Methods Units 1&2

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Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$ü = \frac{1}{2}$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)},\ \lvert r\rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

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Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

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Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	

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