Neap

Trial Examination 2022

Question and Response Booklet

QCE Mathematical Methods Units 3&4

Paper 2 – Technology-active

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 10 short response questions

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2022 QCE Mathematical Methods Units 3&4 Written Examination.

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

Determine the indefinite integrals below.

a) $\int 2\sin(x+4)dx$ *[1 mark]*

b) $\int \frac{1}{2x-9}dx$ *[1 mark]*

c) $\int \frac{1}{3}e^{x}dx$ *[2 marks]*

QUESTION 12 (4 marks)

To receive an invitation to join the SMAAT Club, an adult must achieve a minimum score of 130 in the official SMAAT intelligence test. Assume that the mean score is 100 and the standard deviation is 15 if a random adult from the Australian population were to take the exam.

a) Determine whether scoring within the top 4% of the population is sufficient to receive an invitation to join the SMAAT Club. *[3 marks]*

b) Determine the range of scores that an individual would need to achieve to score within the middle 50% of the population. *[1 mark]*

QUESTION 13 (5 marks)

A factory produces hand-crafted glass ornaments. 1.3% of the ornaments are broken by the time they are packaged in boxes and ready for shipping. Each box of ornaments contains six ornaments.

a) Determine the probability that **one** ornament in a randomly selected box will be broken. *[1 mark]*

b) Determine the probability that **at least one** ornament in a randomly selected box will be broken. *[2 marks]*

c) A quality assurance officer inspects 50 boxes for broken ornaments.

Using your answer for 13b), determine the expected number of boxes that will have **at least one** broken ornament in them. *[2 marks]*

QUESTION 14 (4 marks)

Determine $f(x)$ if $f'(x) = 15e^{9x-4}$ and $f(0) = 1$.

QUESTION 15 (5 marks)

The exotic gypsy moth is an invasive species that defoliates forests in Australia.

A team of environmental scientists conducted a study in a Brisbane forest. They examined 80 randomly selected eucalyptus trees and observed that 13 trees had exotic gypsy moths.

Assume that the sample proportion is normally distributed.

a) Calculate the standard deviation of the sample proportion. *[2 marks]*

b) The sample count is the number of trees counted with the moths if 80 trees were examined.

Calculate the standard deviation of the sample count. *[1 mark]*

c) Based on previous experimental data, the team predicted that 11.30% of trees in the forest would have the moth. Assume that this prediction is the true population proportion.

Calculate the probability of obtaining a sample proportion greater than the proportion calculated in 15a). *[2 marks]*

QUESTION 16 (5 marks)

Consider the equation.

$$f(x) = \frac{1}{kx + c}$$

If $f(x)$ is a continuous probability density function with domain [0, 1], determine the value of c in terms of k.

QUESTION 17 (5 marks)

A curve has the following equation.

$$f(x) = x^3 - 2x^2 - x + 2$$

Determine the area of the region bounded by the curve $f(x)$ and the x-axis.

QUESTION 18 (5 marks)

Diagnostic testing for a virus test is not always accurate. A study of 400 subjects is conducted to test the accuracy of a diagnostic test for dengue fever. The results are shown in the table.

	Positive test	**Negative test**
Has dengue fever	18	4
Does NOT have dengue fever	31	347

a) Calculate an 85% confidence interval of getting an accurate test result. *[3 marks]*

b) A confidence interval with a margin of error of 0.026 was calculated.
Determine the percentage of the confidence interval. *[2 marks]*

QUESTION 19 (7 marks)

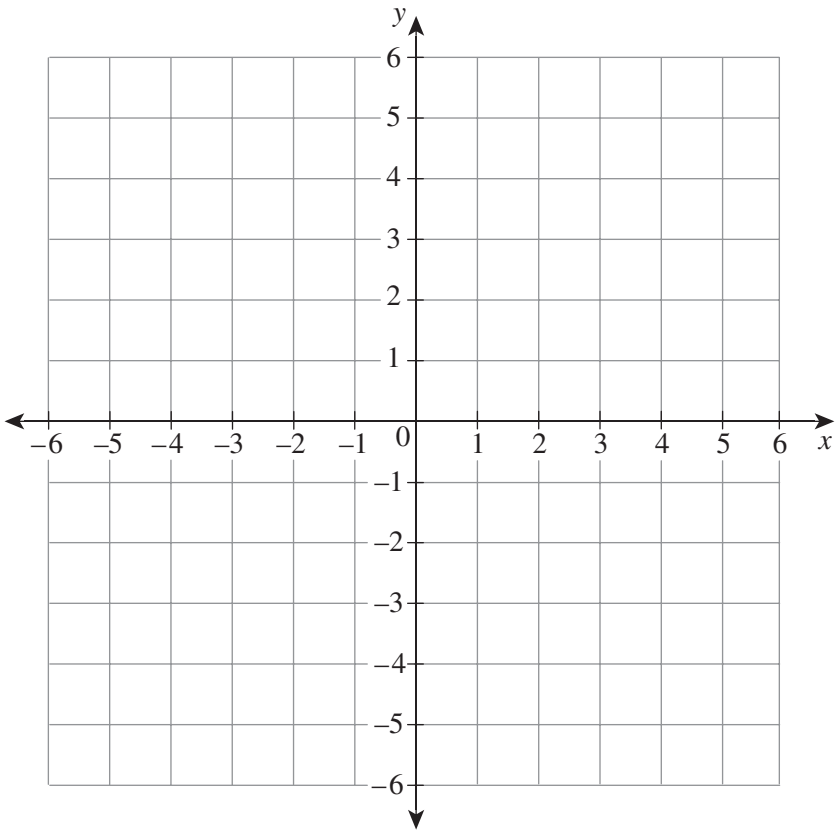
A derivative is represented by the following equation.

$$f'(x) = (x-4)(x-2)^2(x+3)\ln(x)$$

a) Identify all stationary points for $f(x)$ and classify each point. In your answer, provide justification for each stationary point classification. *[4 marks]*

b) Assume that $f(1) = 3$.

On the axes below, draw the graph for $f(x)$. *[3 marks]*



QUESTION 20 (6 marks)

Chlorophyll *a* is a green pigment found in plants. It can be used as a measure of phytoplankton abundance in the ocean. Using experimental data collected in the twenty-first century, scientists have developed the following model to describe the abundance of chlorophyll *a* near Magpie Island since 2000.

$$C(t) = -\frac{1}{7}(t-100)\ln(t+30),$$

where $C(t)$ is the concentration of chlorophyll *a* (in mg per cm^2) near Magpie Island and t is the number of years since January 2000.

In addition, salps, which are an aquatic species, have been studied in the region. It has been found that the salp population depends on the amount of chlorophyll *a* in the region. The following model was developed to represent the relationship.

$$S(x) = -39\cos\left(\frac{\pi}{50}x\right) + 40,$$

where $S(x)$ is the number of individuals per m^2 and x is the amount of chlorophyll *a* (in mg per cm^2).

a) Using the information provided, predict the rate of change in the salp population with respect to time in January 2030. *[4 marks]*

b) State the year and the rate of change when the salp population is decreasing most rapidly with respect to time. Only consider years between 2000 and 2100. *[2 marks]*

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Trial Examination 2022

Formula Booklet

QCE Mathematical Methods Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	