Neap

Trial Examination 2022

Question and Response Booklet

QCE Mathematical Methods Units 3&4

Paper 1 – Technology-free

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are not permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2022 QCE Mathematical Methods Units 3&4 Written Examination.

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (3 marks)

Clara scored 81% in her Chemistry exam and 76% in her Biology exam. The state mean for Chemistry is 51%, and the standard deviation is 15%. The state mean for Biology is 58%, and the standard deviation is 12%. Both exam percentages are normally distributed.

Determine which subject Clara scored better in when compared to the rest of the state.

QUESTION 12 (7 marks)

Calculate the following derivatives and simplify the expressions where possible by collecting like terms, factorising or simplifying fractions.

a) $\frac{d}{dx}\left(\sin(x)\times\left(2x^2-3x\right)+\cos(x)\right)$ *[3 marks]*

b) $\frac{d}{dx}\left(\frac{3\ln(x)}{e^x}\right)$ *[2 marks]*

c) $\frac{d}{dx}\left(\cos^3(x)+9\right)$ *[2 marks]*

QUESTION 13 (4 marks)

An online toy retailer sells approximately 80% of its products from its Top 10 Products list. Assume that this percentage is independent of factors such as time of day and number of products purchased by the same customer.

a) Determine the expected value for a single-item order with respect to whether it will be one of the Top 10 Products or not. *[1 mark]*

b) Determine the variance for a single-item order with respect to whether it will be one of the Top 10 Products or not. *[1 mark]*

c) Determine the probability that **at least one** of the next three products sold will come from the Top 10 Product list. Express your answer using index notation. *[2 marks]*

QUESTION 14 (5 marks)

The displacement of a particle has been modelled using the function $f(t) = 4t^3 - 2t^2 + 3e^{2t}$.

a) Determine the velocity of the particle at $t = 1$. *[2 marks]*

b) Determine the acceleration of the particle at $t = \ln 2$. Express your answer in simplified form. *[3 marks]*

QUESTION 15 (6 marks)

a) Solve $\ln(x) + \ln(x+1) = \ln(2)$. *[3 marks]*

b) Solve $2^{5x+1} + 2^{5x+5} = 17$. *[3 marks]*

QUESTION 16 (8 marks)

For each of the following functions, determine the value of the derivative at $x = 0$.

a) $f(x) = \ln\left(\cos(3x + \pi)\right)$ *[2 marks]*

b) $g(x) = \cos(3x + \pi) \times \ln(x + 1)$ *[2 marks]*

c) $h(x) = \dfrac{\ln(x + 1)}{\cos(3x + \pi)}$ *[3 marks]*

d) Let $j(x) = f(x) - g(x) + h(x)$.
Use your answers for 16a), 16b) and 16c) to determine the value of $j'(0)$. *[1 mark]*

QUESTION 17 (5 marks)

The following information is known.

- $f''(x) = -\frac{4}{x^2}$
- $f'(1) = 5$
- $f(e) = e$

Using the information provided, determine the equation of $f(x)$.

QUESTION 18 (5 marks)

In the context of commercial fisheries, the maximum sustainable yield is defined as the maximum harvest rate (in tonnes of fish harvested per year) such that the fish population remains sustainable. It is calculated by determining the maximum of the function $G(x)$.

A commercial fishery models their fish population using the equation

$$G(x) = 1.3x\left(1 - \frac{x}{k}\right),$$

where G represents the growth rate of fish (tonnes of fish per year), x represents the number of tonnes of fish in the fishery and k represents the carrying capacity.

a) The maximum sustainable yield for the fishery is 4 tonnes per year.

Determine the value of the parameter k. *[3 marks]*

b) Use your answer for 18a) to verify that $x = 4$ maximises $G(x)$. *[2 marks]*

QUESTION 19 (7 marks)

A particle moves along a line. Its velocity v (in m s^{-1}) at time t (in seconds) is modelled by the following piecewise function.

$$v(t) = \begin{cases} \cos\left(\dfrac{\pi(t-4)}{12}\right), & 0 \le t \le 10 \\ -\dfrac{1}{5}t + 2, & 10 < t \le 20 \end{cases}$$

Determine the displacement of the particle during the period $0 \le t \le 20$. Express your answer using exact values.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula Booklet

QCE Mathematical Methods Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	