Neap

Trial Examination 2022

Question and Response Booklet

QCE Mathematical Methods Units 1&2

Paper 2 – Technology-active

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator permitted.
- Formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 11 short response questions

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 11 questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

Use calculus to determine the location and nature of the two stationary points on the function $y = x^3 + 3x^2 - 9x + 2$.

QUESTION 12 (6 marks)

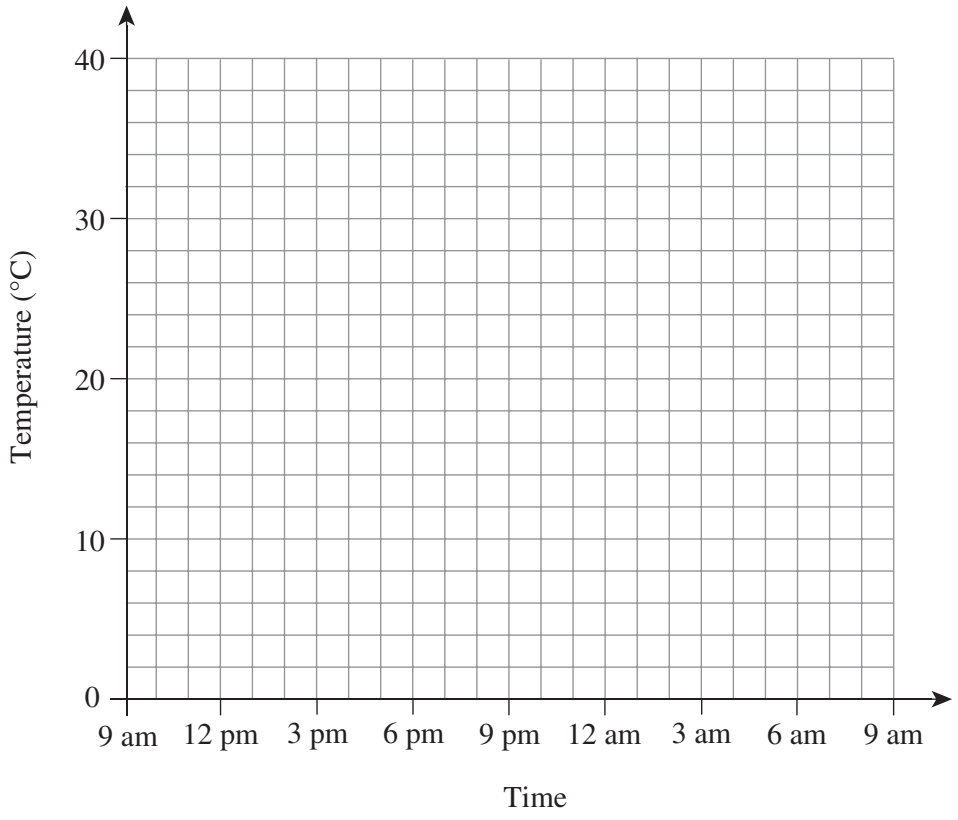
The temperature at a weather station in Queensland is measured over the period of one day. It can be represented by the function

$$T = 9\sin\left(\frac{\pi}{12}t\right) + 25,$$

where T is the temperature in degrees and t is the time in hours measured from 9.00 am.

a) State the maximum temperature and the time that it occurs. *[2 marks]*

b) Sketch the function on the axes provided. *[2 marks]*



c) For how many hours is the temperature above 30°C over the period of the whole day? *[2 marks]*

QUESTION 13 (4 marks)

A ball is dropped and bounces on the ground such that the height reached after each bounce is 35% lower than the previous height. The ball is initially dropped from a position of 2.2 m above the ground.

Determine the peak height of the ball after the fifth bounce.

QUESTION 14 (4 marks)

a) Simplify the expression $\frac{x^3y^7}{(xy^3)^2}$. *[2 marks]*

b) Solve the equation $2^{2x} = 2^x + 30$. *[2 marks]*

QUESTION 15 (4 marks)

Consider $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Determine an expression for $f'(x)$ by differentiating, from first principles, the function $f(x) = 2x^2 + 7$.

QUESTION 16 (4 marks)

The possible points earned (outcomes) and associated probabilities in a game of chance are shown in the table, which is incomplete.

Points (X)	0	1	5	20
Probability ($\Pr(X)$)	0.5	0.25		

It is known that $\Pr(X = 5) = 4 \times \Pr(X = 20)$.

a) Determine the $E(X)$. *[2 marks]*

b) Calculate the probability of scoring more than 20 points over two turns. *[2 marks]*

QUESTION 17 (6 marks)

In a factory, three different screws of different lengths (25 mm, 35 mm and 40 mm) are produced. Of the screws produced, 50% are 25 mm, 30% are 35 mm and 20% are 40 mm. Based on testing, it was found that 6% of the 25 mm screws, 8% of the 35 mm screws and 9% of the 40 mm screws were defective.

a) What is the probability that a screw is defective given that it is a 35 mm screw? *[1 mark]*

b) What percentage of screws are not defective overall? *[2 marks]*

c) What is the probability that a screw is 40 mm, given that it is also defective? *[3 marks]*

QUESTION 18 (6 marks)

The height that a high jumper can reach in any single jump can be modelled as a quadratic equation with respect to time, where time is measured in seconds from the beginning of the jump.

One professional high jumper had a personal best jump of 1.76 m. In a particular jump, she reaches a maximum height that is 8 cm above her personal best. She reaches this height 0.43 seconds into the jump and lands on a padded mat 0.76 seconds after the jump begins.

a) How tall is the mat? *[5 marks]*

b) Evaluate the reasonableness of your solution to 18a). *[1 mark]*

QUESTION 19 (4 marks)

When the expression $(1 + kx)^5$ is expanded, the sum of the coefficients is –1.

Determine the value(s) of k.

QUESTION 20 (5 marks)

At the start of 2010, the number of penguins in colony A was 1200. At the start of 2022, there were 1760 penguins in colony A.

At the start of 2013, the number of penguins in colony B was 800. At the start of 2022, there were 1570 penguins in colony B.

Assuming that the population of penguins can be modelled exponentially, what is the first year that colony B will have twice as many penguins as colony A?

QUESTION 21 (3 marks)

Two hoses are being used to fill a tank with water. Hose one is releasing water into the tank at a rate of $\left(4+\frac{4}{t+1}\right)$ litres per minute and hose two is releasing water into the tank at a rate of $\left(6+\frac{t^2}{t+3}\right)$ litres per minute, where t is the time measured in minutes and $t \geq 0$.

Determine the minimum flow into the tank and the time that the minimum flow occurs.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Write the question number you are responding to.

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Formula Booklet

QCE Mathematical Methods Units 1&2

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	