Neap

Trial Examination 2022

Suggested Solutions

QCE Mathematical Methods Units 1&2

Paper 2 – Technology-active

SECTION 1 – MULTIPLE CHOICE QUESTIONS

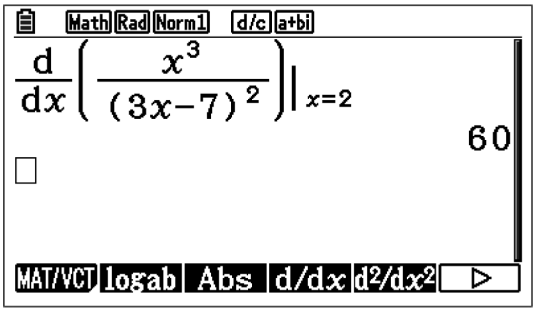
	A	B	C	D
1.	○	○	●	○
2.	○	○	○	●
3.	○	○	○	●
4.	○	○	●	○
5.	●	○	○	○
6.	○	●	○	○
7.	○	○	○	●
8.	○	●	○	○
9.	○	○	●	○
10.	○	○	●	○

QUESTION 1 C

$$
\begin{aligned}
t_n &= t_1 + (n-1)d \\
-64 &= 20 + (n-1)\times(-3) \\
-84 &= -3n + 3 \\
-87 &= -3n \\
n &= 29
\end{aligned}
$$

QUESTION 2 D

$$
\begin{aligned}
f(x) &= \frac{x^2}{(3x-7)^2} \\
f'(x) &= \frac{3x^2\times(3x-7)^2 - 6(3x-7)\times x^3}{((3x-7)^2)^2} \\
f'(2) &= \frac{3(2)^2\times(3(2)-7)^2 - 6(3(2)-7)\times(2)^3}{((3(2)-7)^2)^2} \\
&= \frac{12 - -48}{1} \\
&= 60
\end{aligned}
$$



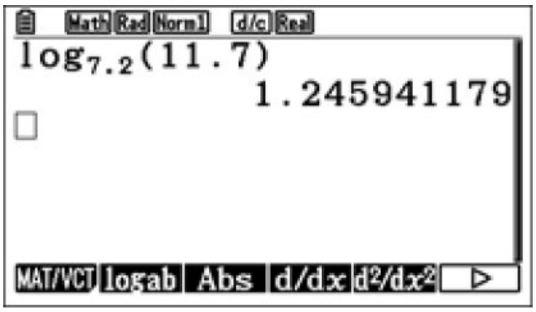
QUESTION 3 D

Using a graphics calculator:

$\log_{7.2} 11.7 = 1.25$

OR

$\frac{\log 11.7}{\log 7.2} = 1.25$



QUESTION 4 C

A graphics calculator may be used to evaluate through trial and error.

Standard square root function:

$y = a\sqrt{x}$

Translating the square root function two units to the right gives:

$y = a\sqrt{x-2}$

Using the coordinates (6, 2) to determine the value of *a* that is the dilation factor gives:

$$\begin{aligned} y &= a\sqrt{x-2} \\ 2 &= a\sqrt{6-2} \\ 2 &= 2a \\ a &= 1 \end{aligned}$$

So, the equation is $y = \sqrt{x-2}$.

QUESTION 5 A

The transformations applied to $f(x) = 3^x$ are

- a horizontal translation by one unit to the right to give $f(x) = 3^{x-1}$
- a vertical dilation by a factor of 2, stretched away from the *x*-axis to give $f(x) = 2 \times 3^{x-1}$
- a reflection over the *x*-axis to give $f(x) = -2 \times 3^{x-1}$.

QUESTION 6 B

$$\begin{aligned} P(46 \text{ or above} \mid \text{Yes}) &= \frac{P(46 \text{ or above} \cap \text{Yes})}{P(\text{Yes})} \\ &= \frac{9}{70} \\ &\approx 0.13 \end{aligned}$$

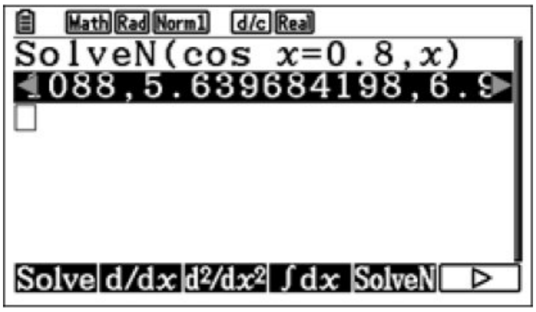
QUESTION 7 D

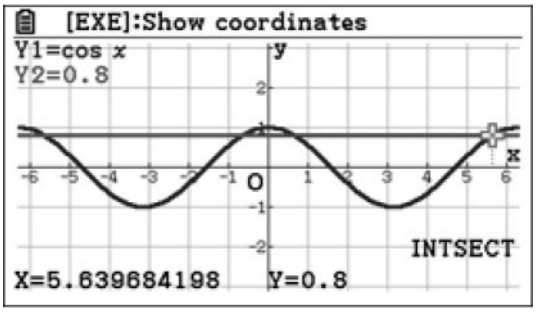
$y = -(x+2)^2 + 4$ is an inverted quadratic with a turning point at (–2, 4).

Thus, the range is $y \le 4$ for all *x*.

QUESTION 8 B

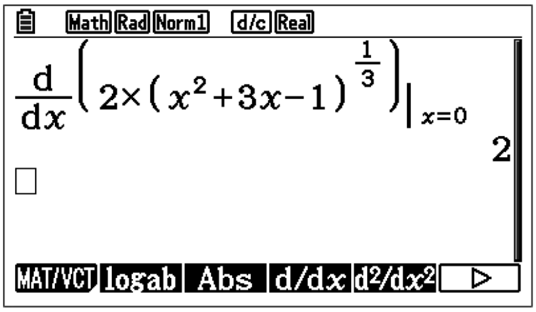
Using a graphics calculator is expected and required, either through SolveN, graph feature or calculator.

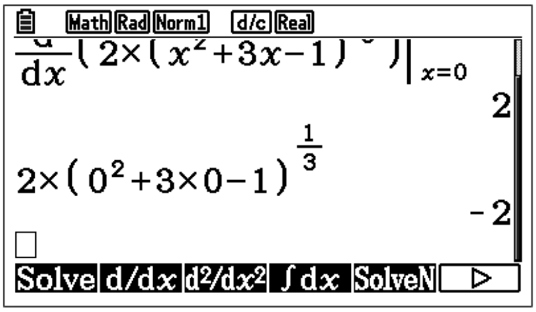




QUESTION 9 C

The graphics calculator can be used to determine the gradient at $x = 0$, and the y-intercept can be found through substitution.





$$f(x) = 2(x^2 + 3x - 1)^{\frac{1}{3}}$$

$$f'(x) = \frac{2}{3}(x^2 + 3x - 1)^{-\frac{2}{3}} \times (2x + 3)$$

$$f'(0) = \frac{2}{3}(-1)^{-\frac{2}{3}} \times (3)$$
$$= 2$$

Therefore, $m = 2$ when $x = 0$. The y-intercept is at $y = -2$.

$y = 2x - 2$

QUESTION 10 C

$$
\begin{aligned}
r^5 &= \frac{t_{11}}{t_6} \\
&= \frac{37}{17} \\
r &= \sqrt[5]{\frac{37}{17}}
\end{aligned}
$$

$$
\begin{aligned}
t_{15} &= t_{11} \times r^4 \\
&= 68.93
\end{aligned}
$$

SECTION 2

QUESTION 11 (4 marks)

$y = x^3 + 3x^2 - 9x + 2$

$y' = 3x^2 + 6x - 9$

$y'' = 6x + 6$

Let $y' = 0$.

$3x^2 + 6x - 9 = 0$

$3(x^2 + 2x - 3) = 0$

$3(x+3)(x-1) = 0$

$x = -3$ and $x = 1$

The stationary points are at $x = -3$ and $x = 1$.

At $x = -3$, $y = (-3)^3 + 3(-3)^2 - 9(-3) + 2 = 29$ and $y'' = 6(-3) + 6 = -12$.

Therefore, there is a maximum at (–3, 29).

At $x = 1$, $y = (1)^3 + 3(1)^2 - 9(1) + 2 = -3$ and $y'' = 6(1) + 6 = 12$.

Therefore, there is a minimum at (1, –3).

[4 marks]
1 mark finding y' and setting $y' = 0$.
1 mark for providing the correct x-values of stationary points.
1 mark for identifying the full coordinates of both stationary points.
1 mark for identifying the nature of both stationary points.
Note: Allow gradient table test to determine the nature of the stationary points.

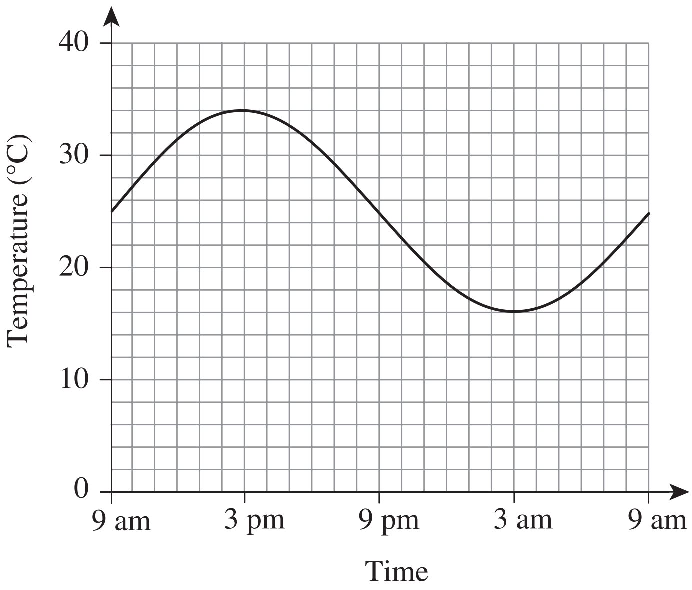
QUESTION 12 (6 marks)

a) The maximum temperature is 34°C.

This occurs at 3 pm.

[2 marks]
1 mark for providing the correct temperature.
1 mark for providing the correct time.

b)

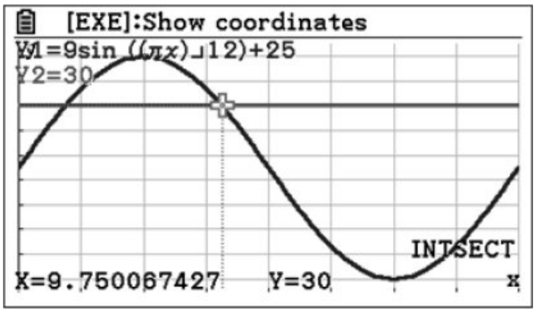


[2 marks]
1 mark for showing the correct amplitude and midline on the graph through peaks and troughs.
1 mark for providing an appropriate period and a neatly sketched graph.

c) First intersection at 2.2499, second intersection at 9.7501.

total time above 30°C $= 9.7501 - 2.2499$

$= 7.5$ hours



[2 marks]
1 mark for identifying the points where the temperature is equal to 30°C. Note: This may be implied by subsequent working.
1 mark for stating that the temperature is above 30°C for 7.5 hours.

QUESTION 13 (4 marks)

$h_5 = h_0 \times r^5$

The decrease in height by 35% corresponds to a common ratio of $r = (1 - 0.35) = 0.65$.

$h_0 = 2.2$

$$\begin{aligned} h_5 &= h_0 \times r^5 \\ &= 2.2 \times 0.65^5 \\ &= 0.255264\ldots \end{aligned}$$

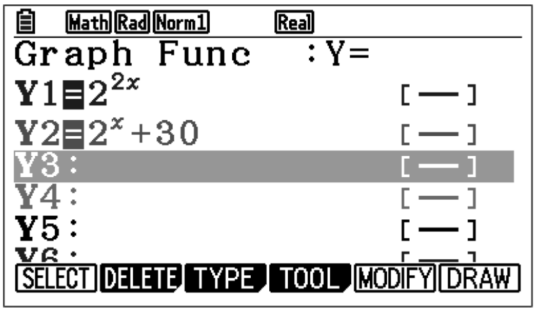
[4 marks]
1 mark for providing an indication of geometric interpretation (may include diagrammatical representation).
1 mark for identifying $h_0 = 2.2$ and $r = 0.65$ Note: This may be implied by subsequent working.
1 mark for using the correct formula to calculate h_0.
1 mark for providing the correct answer.
Note: Accept any appropriate rounding; units are not required.

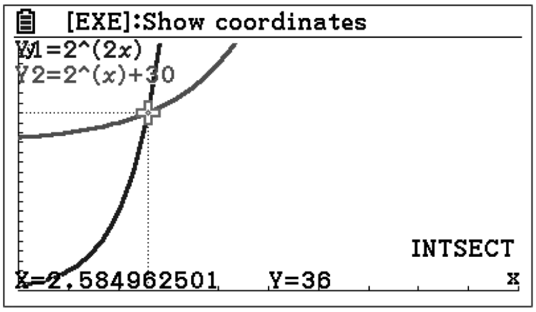
QUESTION 14 (4 marks)

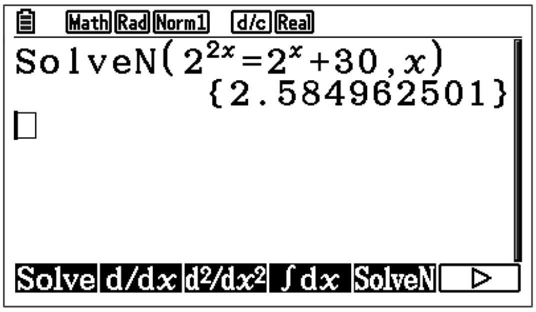
a)
$$\begin{aligned} \frac{x^3y^7}{(xy^3)^2} &= \frac{x^3y^7}{x^2y^6} \\ &= x^{3-2}y^{7-6} \\ &= xy \end{aligned}$$

[2 marks]
1 mark for expanding the brackets in the denominator.
1 mark for providing the correct answer.

b) **Method 1:**







$x = 2.5849\ldots$

[2 marks]

2 marks for providing the correct answer derived through technology.
Note: This can be implied through the statement of the correct answer only.
Accept answers rounded to at least one decimal place.

Method 2:

$$2^{2x} = 2^x + 30$$
$$(2^x)^2 - (2^x) - 30 = 0$$
$$(2^x - 6)(2^x + 5) = 0$$
$$2^x = 6 \text{ (or } -5\text{, an exponential cannot be negative)}$$
$$x = \log_2 6$$
$$x = 2.58496$$

[2 marks]

1 mark for factorising a quadratic in 2^x. Note: This may be implied by subsequent working.
1 mark for providing the correct single value of x.
Note: Accept answers in exact form or rounded to at least one decimal place.

QUESTION 15 (4 marks)

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{(x+h)-x}\right)$$
$$= \lim_{h \to 0}\left(\frac{2(x+h)^2-(2x^2+7)}{h}\right)$$
$$= \lim_{h \to 0}\left(\frac{2(x^2+2xh+h^2)+7-2x^2-7)}{h}\right)$$
$$= \lim_{h \to 0}\left(\frac{2x^2+4xh+2h^2-2x^2}{h}\right)$$
$$= \lim_{h \to 0}\left(\frac{4xh+2h^2}{h}\right)$$
$$= \lim_{h \to 0}(4x+2h)$$
$$= 4x$$

[4 marks]
1 mark for using the gradient formula and x values (x – h and x or x and x + h).
1 mark for making progress towards the simplification up to and including the fourth line of the solution, including the complete expansion of the numerator. Note: This may be implied by subsequent working.
1 mark for completely simplifying to the final solution.
1 mark for showing logical organisation and communication of key steps up to at least the fourth line.

QUESTION 16 (4 marks)

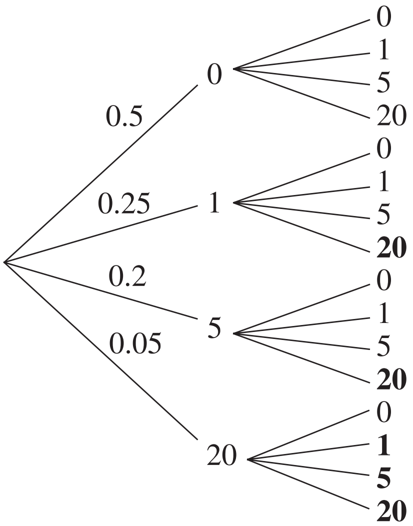
a)
$$\Pr(0)+\Pr(1)+\Pr(5)+\Pr(20)=1$$
$$0.5+0.25+4\times\Pr(20)+\Pr(20)=1$$
$$5\times\Pr(20)=0.25$$
$$\Pr(20)=0.05$$
$$\Pr(5)=0.2$$
$$E(X)=0\times0.5+1\times0.25+5\times0.2+20\times0.05$$
$$=2.25$$

[2 marks]
1 mark for providing the correct Pr(5) and Pr(20) values.
1 mark for providing the correct expected number of points.

b) **Method 1:**

$$\Pr(X > 20) = \Pr(20 \cap 20) + \Pr(20 \cap 5) + \Pr(20 \cap 1)$$
$$= \Pr(20) \times \Pr(20) + 2 \times \Pr(20) \times \Pr(5) + 2 \times \Pr(20) \times \Pr(1)$$
$$= 0.05 \times 0.05 + 2 \times 0.05 \times 0.2 + 2 \times 0.05 \times 0.25$$
$$= 0.0475$$

Method 2:



$1 + 20 = 21;\ 0.25 \times 0.05 = 0.0125$

$5 + 20 = 25;\ 0.2 \times 0.05 = 0.01$

$20 + 1 = 21;\ 0.05 \times 0.25 = 0.0125$

$20 + 5 = 25;\ 0.05 \times 0.2 = 0.01$

$20 + 20 = 40;\ 0.05 \times 0.05 = 0.0025$

$\Pr(X > 20) = 0.0475$

[2 marks]
1 mark for providing a probability statement or a marked-up tree diagram.
1 mark for providing the correct solution.

QUESTION 17 (6 marks)

a) $\Pr(D \mid 35 \text{ mm}) = 0.08$

[1 mark]
1 mark for providing the correct answer.

b) $$\Pr(D') = \Pr(D' \cap 25 \text{ mm}) + \Pr(D' \cap 35 \text{ mm}) + \Pr(D' \cap 40 \text{ mm})$$
$$= \Pr(D') \times \Pr(25 \text{ mm}) + \Pr(D') \times \Pr(35 \text{ mm}) + \Pr(D') \times \Pr(40 \text{ mm})$$
$$= 0.94 \times 0.5 + 0.92 \times 0.3 + 0.91 \times 0.2$$
$$= 0.928$$

[2 marks]
1 mark for providing the correct interpretation (may come as the first step after a tree diagram).
1 mark for providing the correct answer.

c) $$\Pr(40\text{ mm} \mid D) = \frac{\Pr(40\text{ mm} \cap D)}{\Pr(D)}$$
$$= \frac{0.2 \times 0.09}{1 - 0.928}$$
$$= 0.25$$

[3 marks]
1 mark for providing the correct interpretation using the conditional probability rule.
1 mark for using the answer from part b), or otherwise, to determine Pr(D).
1 mark for providing the correct answer.

QUESTION 18 (6 marks)

a) Let $f(t)$ be the high jumper's height in metres and t be the time from the initial jump in seconds.

Consider a quadratic equation in turning point form.

$f(t) = a(t - h)^2 + k$ and (h, k) is the turning point.

$(h, k) = (0.43, 1.84)$, where k is the maximum height reached $(1.76 + 0.08 = 1.84$ cm$)$ at $t = 0.43$ s.

$f(t) = a(t - 0.43)^2 + 1.84$

Substitute $(0, 0)$ as the assumed initial conditions, which is when the high jumper is on the ground at $t = 0$, to find a.

$$0 = a(0 - 0.43)^2 + 1.84$$
$$a = -\frac{1.84}{(0.43)^2}$$
$$a \approx -9.951325...$$

$$f(t) = -\frac{1.84}{0.43^2}(t - 0.43)^2 + 1.84$$

Determine $f(0.76)$:

$$f(0.76) = -\frac{1.84}{0.43^2}(t - 0.43)^2 + 1.84$$
$$\approx 0.756300\ldots$$

The mat is approximately 76 cm tall.

[5 marks]
1 mark for interpreting and suitably identifying a model to apply (turning point form) and defining the variables. Note: This may be implied by subsequent working.
1 mark for substituting (h, k) into the model.
1 mark for substituting (0, 0) into the model to find a.
1 mark for calculating the value of a in exact or decimal form.
1 mark for providing the height of the mat.

b) *Any one of:*

- The solution is reasonable based on the expected height of the mat. It is also reasonable based on the parameters of the jump (or graph).
- The model could be considered unreasonable due to the shape and jumping style of the athlete; the lowest point of the body would move in a more irregular shape as the jumper transitions their body over the bar. Modelling a jump would require a more complex approach.

[1 mark]

1 mark for providing a relevant and suitable comment on the reasonableness of the result.

QUESTION 19 (4 marks)

$$(1+kx)^5 = 1+5kx+10(kx)^2+10(kx)^3+(kx)^4+(kx)^5$$

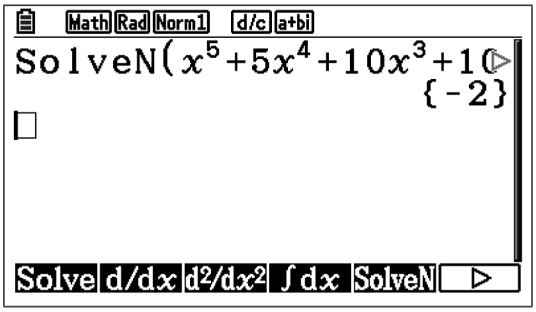
$$= 1+5kx+10k^2x^2+10k^3x^3+5k^4x^4+k^5x^5$$

Given that the sum of the coefficients is –1:

$$1+5k+10k^2+10k^3+5k^4+k^5=-1$$

$$k^5+5k^4+10k^3+10k^2+5k+2=0$$

Using a graphics calculator:



$k = -2$

[4 marks]

1 mark for providing a binomial expansion including four correct terms. Note: This may be implied by subsequent working.

1 mark for providing the correct expansion of the binomial. Note: This may be implied by subsequent working.

1 mark for providing a correct expression for the sum of coefficients.

1 mark for providing the correct answer.

Note: Accept a solution of –1 that is achieved when the constant is not included.

QUESTION 20 (5 marks)

Let t = time (years) from 2010 and P_A and P_B be the populations of penguins in colonies A and B, respectively.

Method 1 (using technology):

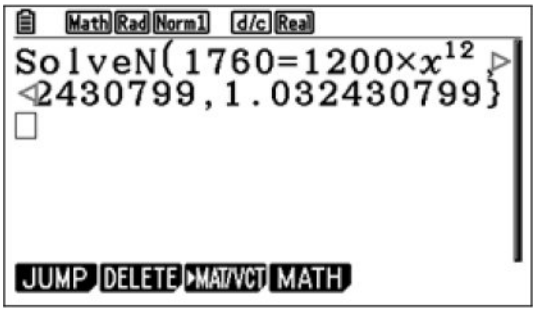
For colony A:

$P_A = A \times b^t$

Known values:

$t = 0,\ P_A = 1200 \ \therefore A = 1200$

$t = 12,\ P_A = 1760$



$P_A = 1200 \times 1.0324^t$

For colony B:

$P_B = A \times b^t$

Known values:

$t = 3,\ P_A = 800$

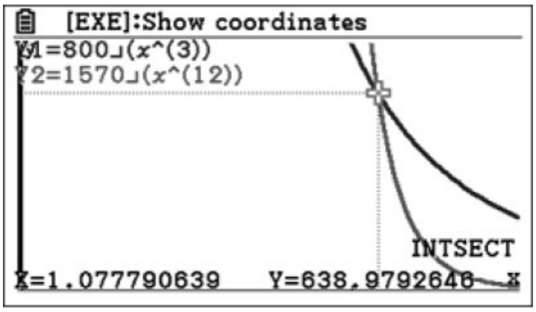
$t = 12,\ P_A = 1570$

$t = 3,\ 800 = A \times b^3$

$t = 12,\ 1760 = A \times b^{12}$

$A = 638.97926\ldots$

$B = 1.07779\ldots$

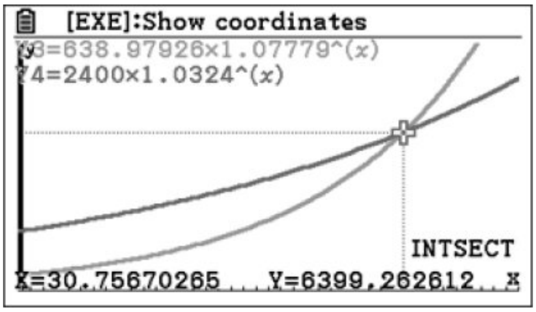


$P_A = 638.97926 \times 1.07779^t$

Final solution:

Let $P_B = 2 \times P_A$.

$638.97926 \times 1.07779^t = 2 \times 1200 \times 1.0324^t$



The population of colony B will be double that of colony A at $t = 30.7567\ldots$ years after 2010; therefore, sometime late in the year 2040.

[5 marks]
1 mark for developing a suitable model for colony A with constants found.
1 mark for developing a suitable model for colony B with constants found.
1 mark for setting up an equation where population of B is equal to double the population of A.
1 mark for providing the correct time, based on the initial variable definitions.
1 mark for converting the time into the corresponding year of 2040, as required by the question.
Note: There are a range of suitable initial parameters that can lead to the correct solution; apply the mark allocations above to any suitable approach. Accept any suitable rounding that leads to an answer in the year 2040.

Method 2 (without technology):

A base value should be chosen; in this case, 2.

For colony A:

$P_A = A \times b^{kt}$, let $b = 2$

$P_A = A \times 2^{kt}$

$P_A = A \times 2^{kt}$, where $t = 0$ and $PA = 1200$

$1200 = A \times 2^0$

$A = 1200$

$P_A = A \times 2^{kt}$, where $t = 12$ and $\mathrm{P}_A = 1760$

$1760 = 1200 \times 2^{k \times 12}$

$2^{k \times 12} = \frac{1760}{1200} \approx 1.46666\ldots$

$12k = \log_2 \frac{1760}{1200}$

$k = \frac{1}{12} \log_2 \frac{1760}{1200}$

≈ 0.046045085

The model for colony A is $P_A = 1200 \times 2^{0.046t}$.

For colony B:

$P_B = A \times b^{kt}$, let $b = 2$

$P_B = A \times 2^{kt}$

Known values: $t = 3$, $P_B = 800$ and $t = 12$, $P_B = 1570$

(1) $800 = A \times 2^{3k}$

(2) $1570 = A \times 2^{12k}$

Divide (2) by (1):

$$\frac{1570}{800} = \frac{A \times 2^{12k}}{A \times 2^{3k}}$$

$$2^{9k} = \frac{1570}{800}$$

$$k = \frac{1}{9}\log_2 \frac{1570}{800}$$

$$\approx 0.1087696...$$

$P_B = A \times 2^{0.108t}$

Calculate A:

$$800 = A \times 2^{0.108 \times 3}$$

$$A = \frac{800}{2^{0.324}}$$

$$= 638.97926\ldots$$

Model for colony B:

$P_B = 638.98 \times 2^{0.108t}$

Determine when $P_B = 2 \times P_A$:

$$638.98 \times 2^{0.108t} = 2 \times 1200 \times 2^{0.046t}$$

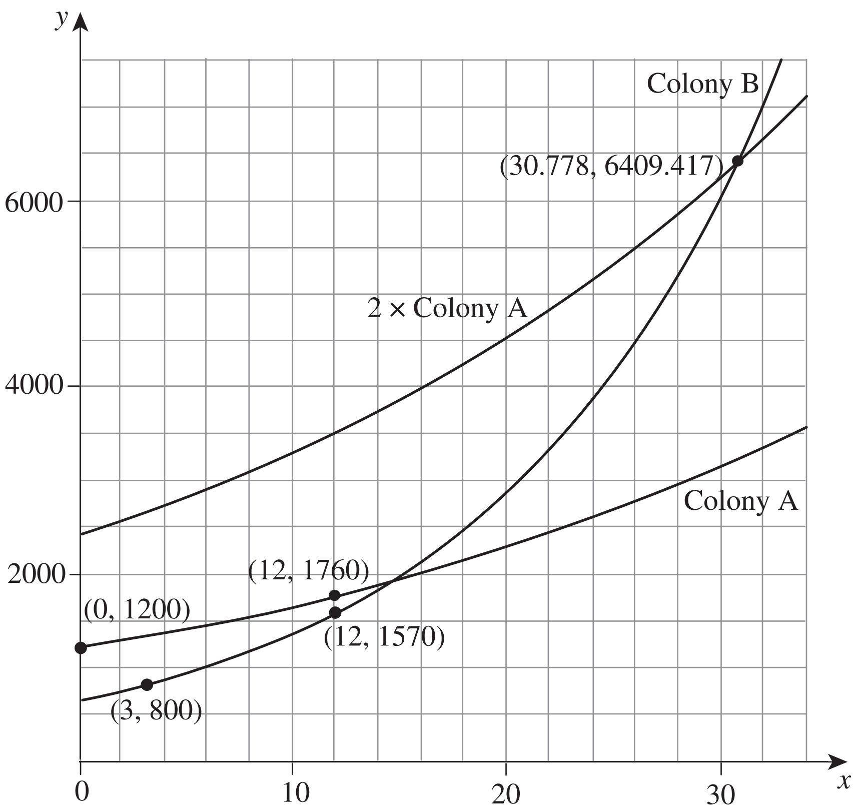
$$\frac{2^{0.108t}}{2^{0.046t}} = \frac{2400}{638.98}$$

$$2^{0.62t} = \frac{2400}{638.98}$$

$$t = \frac{1}{0.62}\log_2 \frac{2400}{638.98}$$

$$t \approx 30.77761781$$

The population of colony B will be double that of colony A at $t = 30.7567\ldots$ years after 2010; therefore, sometime late in the year 2040.



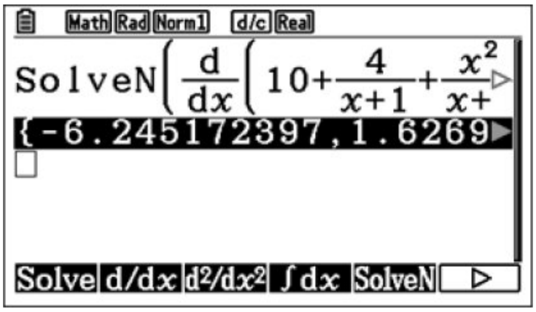
[5 marks]

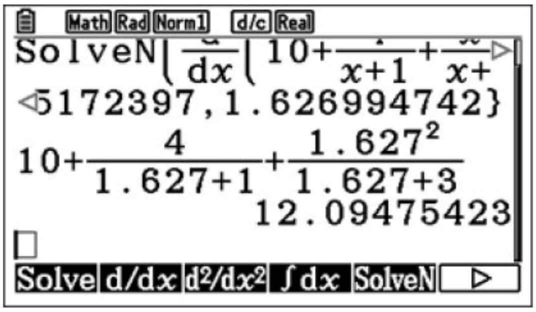
1 mark for development of a suitable model for colony A with constants found.
1 mark for development of a suitable model for colony B with constants found.
1 mark for setting up equation where population of B is equal to double the population of A.
1 mark for the correct time given, based on the initial variable definitions.
1 mark for returning answer into the context provided to identify 2040 as the year.
Note: There are a range of suitable initial parameters that can lead to the correct solution, apply the mark allocations above to any suitable approach. Accept any suitable rounding that leads to an answer in the year 2040. A graph is not required as part of the response.

QUESTION 21 (3 marks)

total flow rate = r

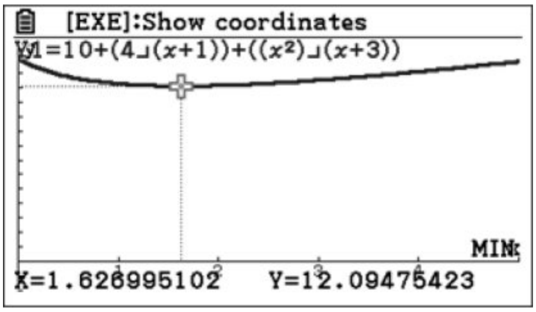
$$r = 10 + \frac{4}{t+1} + \frac{t^2}{t+3}$$





Minimum r = 12.09475… litres per minute at time 1.6269947… minutes (or 1 minute and 37.62 seconds).

Accept graphical method:



[3 marks]

1 mark for developing a correct formula for the overall rate. Note: This may be implied by subsequent working.

1 mark for providing the correct minimum flow (accept any suitable rounding).

1 mark for providing the correct time at which the minimum flow occurs (accept any suitable rounding).

Note: Award marks as shown, irrespective of any algebraic/calculus work developed as part of a solution. Accept a graphical method.

 TEQ MM12_SS_P2_2022