Neap

Trial Examination 2022

Question and Response Booklet

QCE Mathematical Methods Units 1&2

Paper 1 – Technology-free

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are not permitted.
- Formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 10 short response questions

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (3 marks)

Consider the quadratic function $y = a(x - 3)^2 - 5$.

a) State the coordinates of the turning point. *[1 mark]*

b) State the equation of the axis of symmetry. *[1 mark]*

c) Determine the value of a, given that the function has a y-intercept at $y = 1$. *[1 mark]*

QUESTION 12 (5 marks)

The function $f(x)$ is shown.

$$f(x) = x^3 + 2x^2 - 5x - 6$$

Given that $f(-3) = 0$, fully factorise $f(x)$.

QUESTION 13 (3 marks)

An arithmetic sequence is defined as $t_n = 4n - 3$.

a) Calculate the first term of the sequence. *[1 mark]*

b) Determine the common difference. *[1 mark]*

c) Calculate the eighth term of the sequence. *[1 mark]*

QUESTION 14 (6 marks)

Differentiate the following functions.

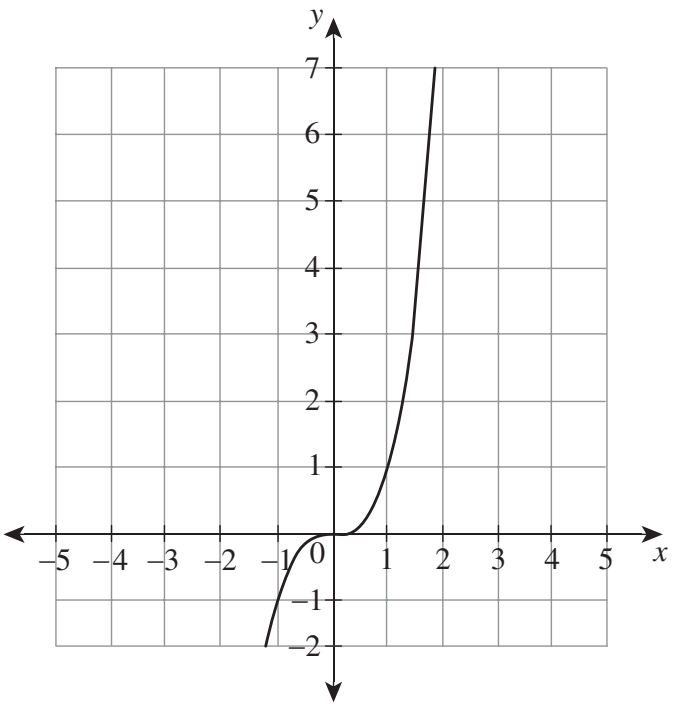
a) $y = 2x^3$ *[1 mark]*

b) $y = (3x - 4)^5$ *[2 marks]*

c) $y = 2x\sqrt{x^2 - 4}$ *[3 marks]*

QUESTION 15 (5 marks)

The graph of $y = x^3$ is shown.



a) On the axes above, sketch the graph of $y = \frac{1}{2}(x-2)^3 + 4$ and state the key points of the graph. *[2 marks]*

b) Describe, in order, the transformations applied to $y = x^3$ to give $y = \frac{1}{2}(x-2)^3 + 4$. *[3 marks]*

QUESTION 16 (4 marks)

A child is playing with a remote-control car. The car's position with regards to the child, x metres (m), at time, t, seconds (s) can be modelled by the function

$$x(t) = 4 + 20t - 5t^2.$$

a) Determine the initial distance of the car from the child. *[1 mark]*

b) Determine a function for the velocity of the car. *[1 mark]*

c) Calculate the furthest position that the car reaches in the positive direction before returning to the child. *[2 marks]*

QUESTION 17 (7 marks)

a) Show that, for all values of k, the equation $y = 2x^2 - kx + k - 3$ has two solutions. *[4 marks]*

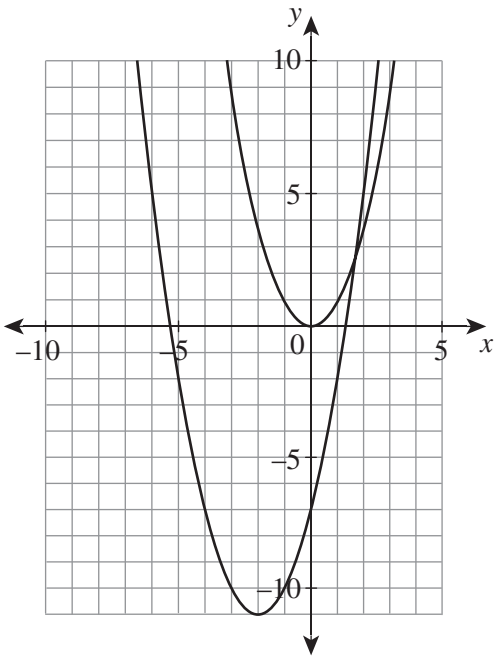
b) If 2 is a solution, determine all possible values of k. *[3 marks]*

QUESTION 18 (5 marks)

Solve the simultaneous equations $4^{2x-1} = 8^{x+y}$ and $27^{3y} = 3^{2x-7}$.

QUESTION 19 (6 marks)

The two quadratic functions $f(x) = x^2$ and $g(x) = x^2 + 4x - 7$ are shown on the graph.



A point is chosen on each function so that the tangents at these points have the same gradient.

Show that the distance between these two points remains constant as their position varies, and determine the exact distance between the two points.

QUESTION 20 (6 marks)

The first three terms of a geometric sequence are $3b - 5$, $2b$ and $b + 6$, respectively, where b is an integer.

Determine all possible sums to infinity of this sequence.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula Booklet

QCE Mathematical Methods Units 1&2

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	