Neap

Trial Examination 2021

Suggested solutions

QCE Mathematical Methods Units 3&4

Paper 2 – Technology-active

SECTION 1 – MULTIPLE-CHOICE QUESTIONS

	A	B	C	D
1.	○	○	●	○
2.	●	○	○	○
3.	○	●	○	○
4.	○	●	○	○
5.	○	●	○	○
6.	○	○	●	○
7.	○	○	○	●
8.	○	○	●	○
9.	○	○	●	○
10.	●	○	○	○

QUESTION 1 C

$$f'(x) = 15e^{\sin(x)} \times \cos(x)$$

$$\begin{aligned} f'(0) &= 15e^{\sin(0)} \times \cos(0) \\ &= 15 \end{aligned}$$

QUESTION 2 A

$$f(x) = -1.2\cos\left(\frac{15\pi x + 31}{93}\right)$$

$$\begin{aligned} f'(x) &= 1.2\sin\left(\frac{15\pi x + 31}{93}\right) \times \frac{15\pi}{93} \\ &= \frac{6\pi}{31}\sin\left(\frac{15\pi x + 31}{93}\right) \end{aligned}$$

$$\begin{aligned} f'(5.5) &= \frac{6\pi}{31}\left(\frac{15\pi \times 5.5 + 31}{93}\right) \\ &= 0.01298855199 \\ &\approx 0.013 \text{ m/h} \end{aligned}$$

QUESTION 3 B

$$\begin{aligned} \text{area} &= \frac{1}{2}bc\sin(A) \\ &= \frac{1}{2} \times 11 \times 11\sin\left(\frac{4\pi}{7}\right) \\ &\approx 59 \text{ units}^2 \end{aligned}$$

QUESTION 4 B

$$f(x) = \text{In}\left(e^x + 2\right)$$

$$f'(x) = \frac{e^x}{e^x + 2}$$

$$f''(x) = \frac{e^x \times \left(e^x + 2\right) - e^x \times e^x}{\left(e^x + 2\right)^2}$$

$$= \frac{2e^x}{\left(e^x + 2\right)^2}$$

$$f''(0) = \frac{2e^0}{\left(e^0 + 2\right)^2}$$

$$= \frac{2}{(1+2)^2}$$

$$= \frac{2}{9}$$

$$\approx 0.222$$

QUESTION 5 B

$$\text{variance} = 0.35(1 - 0.35)$$

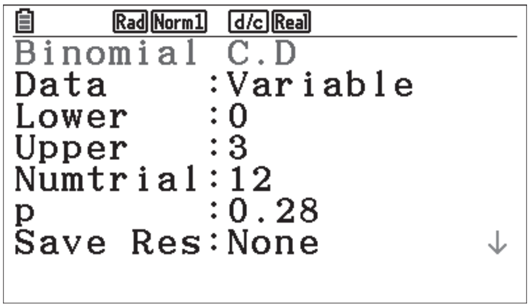
$$= 0.2275$$

$$\text{standard deviation} \approx \sqrt{0.2275}$$

$$\approx 0.48$$

QUESTION 6 C

Using a graphics calculator: Statistics, DIST, BINOMIAL, Bcd.



Using this method, the probability that fewer than 4 cartons will contain an egg stuck to the carton is 0.55.

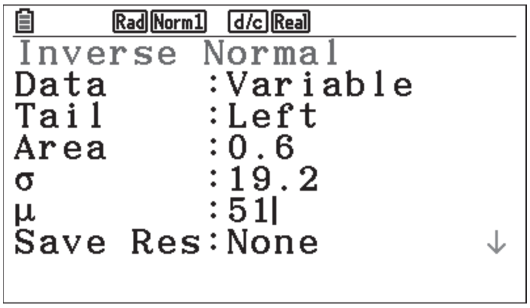
QUESTION 7 D

As $h = fg$, $h' = fg' + f'g$.

$$\begin{aligned} h'(a) &= f(a)g'(a) + f'(a)g(a) \\ &= (5 \times 12) + (-2 \times 7) \\ &= 46 \end{aligned}$$

QUESTION 8 C

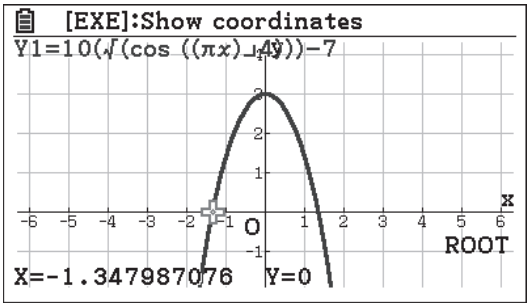
Using a graphics calculator: Statistics, DIST, NORM, InvN.

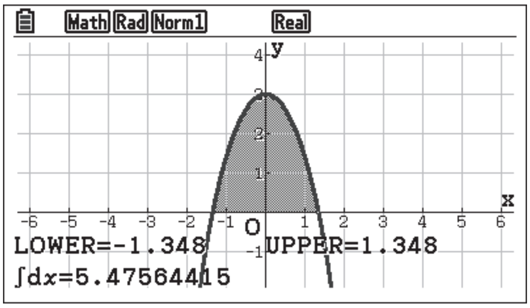


Using this method, the value of $b = 55.86$.

QUESTION 9 C

In graphing the function, the roots are at 1.348 and –1.348. These roots are the bounds of the integration. Integrating over these bounds gives 5.48.





QUESTION 10 A

A is correct. The solution is obtained by using the formula for the mean of a continuous random distribution. **B** is incorrect. x should be in the integral. **C** is incorrect. This option is the integral for the variance, not the mean. **D** is incorrect. The integral is missing an x.

SECTION 2

QUESTION 11 (4 marks)

a) $$\frac{d}{dx}\left(\sin\left(\ln\left(x\right)\right)\right) = \cos\left(\ln\left(x\right)\right) \times \frac{1}{x}$$
$$= \frac{\cos\left(\ln\left(x\right)\right)}{x}$$

[2 marks]
1 mark for using the chain rule correctly.
1 mark for correctly determining the final solution.

b) $$\frac{d}{dx}\left(e^{\pi x} \times \left(3x^2 + 4x + 8\right)\right) = \pi e^{\pi x} \times \left(3x^2 + 4x + 8\right) + e^{\pi x} \times \left(6x + 4\right)$$
$$= e^{\pi x}\left(\pi\left(3x^2 + 4x + 8\right) + \left(6x + 4\right)\right)$$

[2 marks]
1 mark for using the product rule correctly.
1 mark for correctly determining the final solution.

QUESTION 12 (4 marks)

a) $$\int 8x^3 - \frac{1}{x^3} dx = \frac{8x^4}{4} - \frac{\frac{1}{x^2}}{-2} + c$$
$$= 2x^4 + \frac{1}{2x^2} + c$$

alternative form: $= \frac{4x^6 + 1}{2x^2}$

[2 marks]
1 mark for integrating either term.
1 mark for providing the correct solution with the constant of integration.

b) $$\int \sin\left(3 - 5x\right) dx = \frac{-\cos\left(3 - 5x\right)}{-5}$$
$$= \frac{1}{5}\cos\left(3 - 5x\right) + c$$

[2 marks]
1 mark for identifying that the integral of sin *is* –cos.
1 mark for providing the correct solution with the constant of integration.

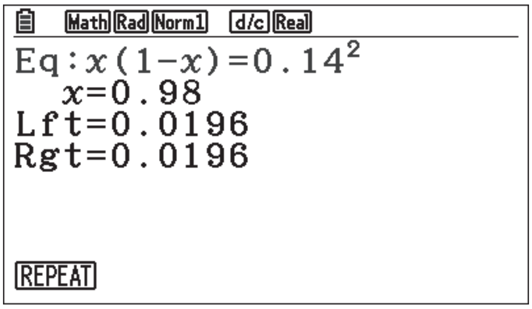
QUESTION 13 (4 marks)

a) This is a Bernoulli distribution.

Thus, variance $= p(1-p)$

$= 0.14^2$

Using a graphics calculator: Equation, Solver.



$p = 0.98$ or $p = 1 - 0.98$

$= 0.02$

Since $p < 0.5$, $p = 0.02$.

[3 marks]
1 mark for using the correct variance OR standard deviation formula.
1 mark for establishing the correct equation.
1 mark for correctly determining the value of p.

b) $\mathrm{E}(X) = p$

$= 0.02$

[1 mark]
1 mark for correctly determining the expected value.
Note: Accept follow-through errors from 13a).

QUESTION 14 (4 marks)

a) mean $= np$

$= \frac{4}{1000} \times 25\,000$

$= 100$

variance $= np(1-p)$

$= 25\,000 \times \frac{4}{1000} \times \frac{996}{1000}$

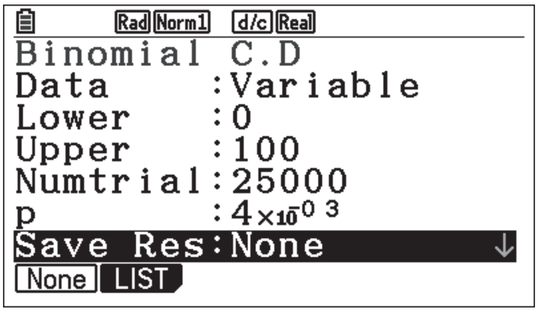
$= 99.6$

standard deviation $= \sqrt{99.6}$

≈ 9.98

[3 marks]
1 mark for correctly calculating the mean.
1 mark for correctly calculating the variance.
1 mark for correctly calculating the standard deviation.
Note: Accept follow-through errors when determining the variance and standard deviation.
Final answer given to two decimal places or more is acceptable.

b) Using a graphics calculator: Statistics, DIST, BINOMIAL, Bcd.



$$p = 0.52656225$$
$$\approx 0.53$$

[1 mark]
1 mark for providing the correct probability.
Note: Final answer given to two decimal places or more is acceptable.

QUESTION 15 (5 marks)

a) $$x'(t) = \frac{18.8e^{2t}}{2} - \frac{2.1}{t+1} \times -1$$
$$= 9.4e^{2t} + \frac{2.1}{t+1} + c$$

Substituting $x'(0) = 11.5$ gives:

$$x'(0) = 9.4e^{2\times 0} + \frac{2.1}{0+1} + c$$
$$= 11.5$$

$$9.4 + 2.1 + c = 11.5$$
$$c = 0$$

$$\therefore x'(t) = 9.4e^{2t} + \frac{2.1}{t+1}$$

[2 marks]
1 mark for correctly integrating the function.
1 mark for correctly determining the constant of integration.

b) $x(t) = \int x'(t)\,dx$

$= x'(t)$

$= \dfrac{9.4e^{2t}}{2} + 2.1\ln|t+1| + d$

$= 4.7e^{2t} + 2.1\ln|t+1| + d$

Substituting $x(0) = 6$ gives:

$4.7e^{2\times 0} + 2.1\ln|0+1| + d = 6$

$d = 1.3$

$\therefore x(t) = 4.7e^{2t} + 2.1\ln|t+1| + 1.3$

[3 marks]
1 mark for correctly integrating the velocity function.
1 mark for using absolute values for the logarithmic function.
1 mark for correctly determining the constant of integration.
Note: Responses do not require absolute values for the velocity function to receive full marks.

QUESTION 16 (5 marks)

a) The probability is 0. This is because the probability of a point of a continuous distribution is 0.

[2 marks]
1 mark for providing the correct probability.
1 mark for providing an appropriate explanation.

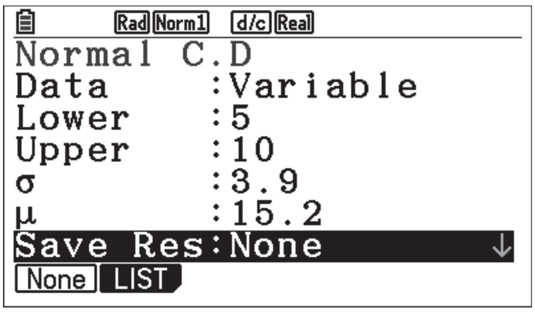
b) As the mean is 15.2 and the normal distribution is symmetrical, the probability is approximately 0.5.

[1 mark]
1 mark for providing the correct probability.

c) approximately 0.95.

[1 mark]
1 mark for providing the correct probability.

d) Using a graphics calculator: Statistics, DIST, NORM, Ncd, inputting the mean, standard deviation and the number of hours required.



$p = 0.08675486$

≈ 0.087

[1 mark]
1 mark for providing the correct probability.
Note: Final answer given to two decimal places or more is acceptable.

QUESTION 17 (5 marks)

a) $p = \frac{47}{200}$

$= 0.235$

[1 mark]

1 mark for correctly determining the sample proportion.

b) estimate of the population standard deviation $= \sqrt{\frac{p(1-p)}{n}}$

$= \sqrt{\frac{0.235(1-0.235)}{200}}$

≈ 0.02998124414

≈ 0.03

[1 mark]

1 mark for correctly estimating the population standard deviation.

Note: Final answer given to two decimal places or more is acceptable.

c) For a 95% confidence interval, $z = 1.96$.

$$\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = (0.235 - 1.96 \times 0.03,\ 0.235 + 1.96 \times 0.03)$$

$$= (0.176,\ 0.294)$$

[3 marks]

1 mark for correctly determining the z-value.

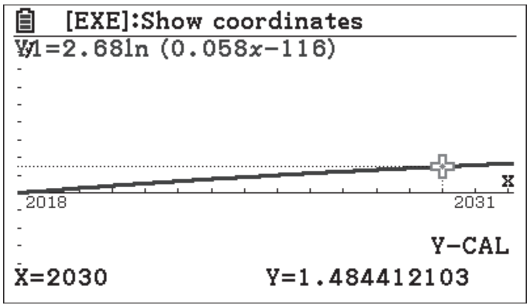
1 mark for using the correct formula for the confidence interval.

1 mark for correctly determining the confidence interval.

Note: Final answer given to two decimal places or more is acceptable.

QUESTION 18 (5 marks)

a) Using a graphics calculator: using plot and inputting the x-value of 2030.



$y \approx 1.484412103$

$\therefore$ number of users $\approx e^{1.484412103}$

≈ 4.41 billion

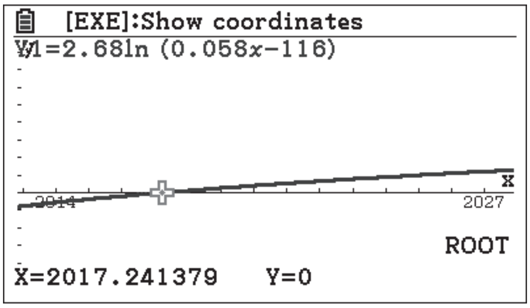
[2 marks]

1 mark for determining the approximate value 1.484412103.

1 mark for interpreting the result correctly to determine the number of users.

Note: Accept answers 4 412 370 632 or 4.41 billion.

b) Using a graphics calculator: G-solve, ROOT (because ln = 0).

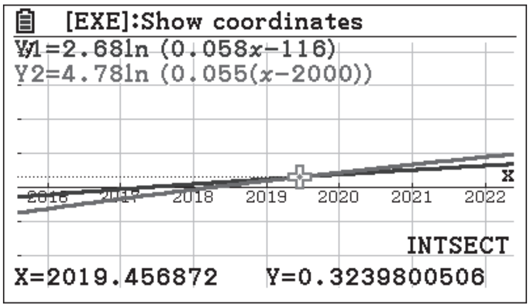


$x \approx 2017.241379$

Therefore, Connexting reached one billion users in 2017.

[1 mark]
1 mark for correctly determining the year.

c) Using a graphics calculator, G-solv, INTSECT.



$x \approx 2019.456872$

Therefore, OPzest and Connexting had the same total number of users in 2019.

[1 mark]
1 mark for correctly determining the year.

d) $\therefore$ number of users $\approx e^{0.3239800506}$

≈ 1.38 billion (to two decimal places)

[1 mark]
1 mark for correctly determining the number of users.

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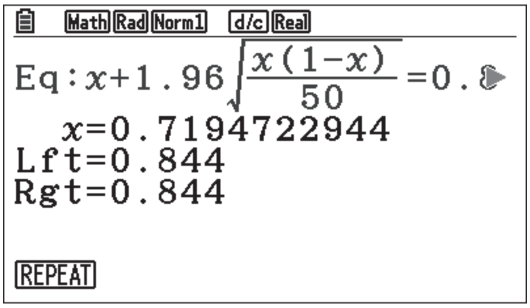
QUESTION 19 (6 marks)

95% confidence interval has a z-score of approximately 1.96. $n = 50$ and the upper bound is 0.844. The margin of error is given by the formula $z = \sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$. The upper bound of a confidence interval is given by the formula $\hat{p} + z\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$. Substituting the information above gives $\hat{p} + 1.96\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{50}} = 0.844$.

Using a graphics calculator: Equation, Solver.



$\hat{p} = 0.719$

$$\text{margin of error} = 1.96\sqrt{\frac{0.719\left(1-0.719\right)}{50}}$$
$$\approx 0.1246$$

$$\text{lower bound} = \hat{p} - 0.1246$$
$$\approx 0.594$$
$$\approx 0.59$$

[6 marks]

1 mark for determining the appropriate z-score.

1 mark for establishing the correct equation.

1 mark for appropriate justification for solving for $\hat{p}$.

1 mark for correctly solving for $\hat{p}$.

1 mark for providing the correct margin of error.

1 mark for correctly determining the lower bound of the confidence interval.

Note: Accept follow-through errors when determining the margin of error and the lower bound. Final answer given to two decimal places or more is acceptable.

QUESTION 20 (8 marks)

Due to continuity, $2bc^2 = bc$.

$$2bc^2 - bc = 0$$
$$bc(2c-1) = 0$$

By Null Factor Law, $c = 0$ or $c = \frac{1}{2}$.

$c = 0$ is rejected, so c must equal $\frac{1}{2}$.

$$f(x) = \begin{cases} 2bx^2, & \frac{1}{2} < x < 1 \\ bx, & 0 < x < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

Being a probability distribution:

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Thus, $\int_0^{\frac{1}{2}} bx\ dx + \int_{\frac{1}{2}}^{1} 2bx^2 = 1$

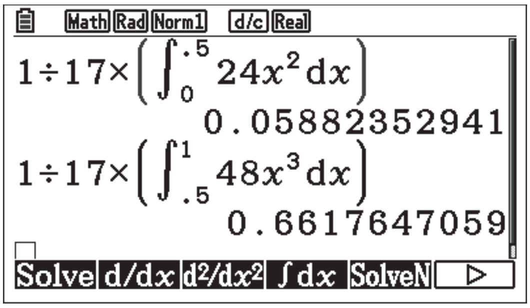
$$\left[\frac{bx^2}{2}\right]_0^{\frac{1}{2}} + \left[\frac{2bx^3}{3}\right]_{\frac{1}{2}}^{1} = 1$$

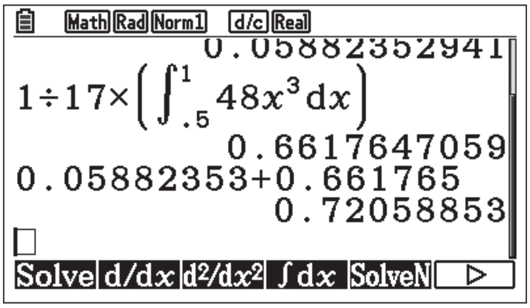
Thus, $\frac{0.5^2 b}{2} + \frac{2b}{3} - \frac{2 \times 0.5^3 b}{3} = 1$

$$\therefore b = \frac{24}{17}$$

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} f(x)x\ dx \\ &= \frac{1}{17}\left(\int_0^{\frac{1}{2}} 24x^2\ dx + \int_{\frac{1}{2}}^{1} 48x^3\right) \end{aligned}$$

Using a graphics calculator: OPTN, CALC, F4 (integration option).





$$E(X) = \frac{49}{68}$$
$$\approx 0.72$$

[8 marks]

1 mark for establishing that $2bx^2 = bx$ at point c.

1 mark for determining that $c = \frac{1}{2}$.

1 mark for recognising that the integral of the probability density function over its domain is 1.

1 mark for establishing the sum of definite integrals that need to be integrated.

1 mark for correctly integrating both integrals.

1 mark for determining the value of b.

1 mark for establishing the expression for determining the expected value.

1 mark for correctly determining the expected value.

Note: Final answer given to two decimal places or more is acceptable for determining the value of b and the expected value.