Neap

Trial Examination 2022

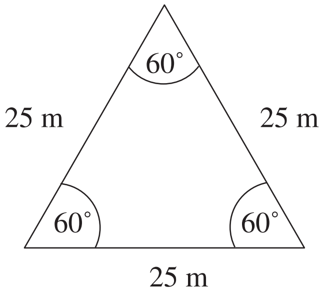
Suggested Solutions

QCE General Mathematics Units 3&4

Paper 2

SECTION 1

QUESTION 1 (3 marks)



$$\begin{aligned}\text{area} &= \frac{1}{2}bc\sin A\\ &= \frac{1}{2}\times 25\times 25\sin(60)\\ &= 270.6329\ \text{m}^2\end{aligned}$$

Let t_0 be 2015.

$t_0 = 8.50$, $t_{n+1} = 1.028 \times t_n$

$t_1 = 8.738$

$t_2 = 8.983$

$t_3 = 9.234$

$t_4 = 9.493$

$t_5 = 9.759$

$t_6 = 10.032$

$t_7 = 10.313$

Therefore, the cost of fertiliser in 2022 is \$10.31 per square metre.

$$\begin{aligned}\text{the cost of preparing the farmer's paddock} &= 270.6329\times 10.31\\ &= \$2790.23\end{aligned}$$

[3 marks]
1 mark for calculating the area of the paddock.
1 mark for calculating the cost of fertiliser in 2022.
Note: Accept alternative methods.
1 mark for calculating the cost of preparing the paddock.

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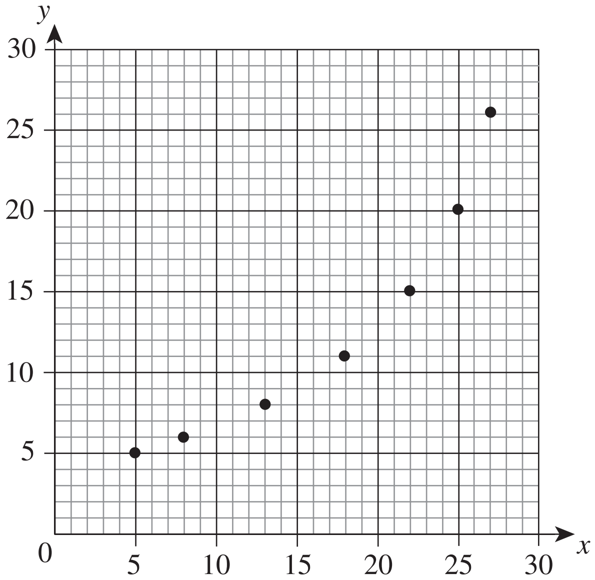
QUESTION 2 (6 marks)



The equation for the least-squares line is $y = -1.594 + 0.866x$.

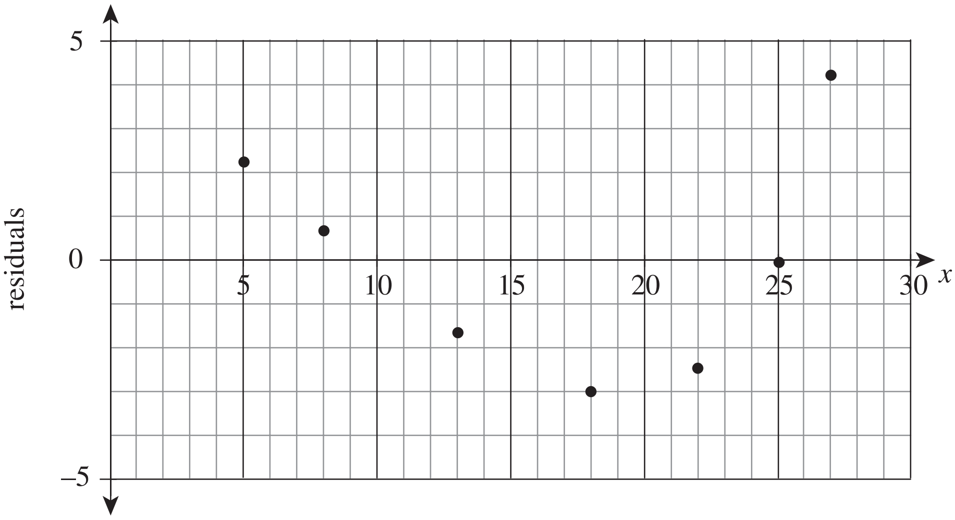
However, on inspection, the graph does not appear linear. A residual plot can evaluate the linearity.

For example:

$$\begin{aligned}\text{predicted value of } y &= -1.594 + 0.866 \times 13 \\ &= 9.664\end{aligned}$$

$$\begin{aligned}\text{residual value} &= \text{actual } y \text{ value} - \text{predicted } y \text{ value} \\ &= 8 - 9.664 \\ &= -1.664\end{aligned}$$

x	*y*	**Predicted *y* value**	**Residual *y* value**
5	5	2.736	2.264
8	6	5.334	0.666
13	8	9.664	–1.664
18	11	13.994	–2.994
22	15	17.458	–2.458
25	20	20.056	–0.056
27	26	21.788	4.212



Upon evaluation, the residual plot has a clear curved pattern across the x-axis, which suggests the presence of a non-linear association between x and y.

[6 marks]
1 mark for drawing a scatterplot.
1 mark for determining the least-squares line equation.
1 mark for calculating the predicted values of y.
1 mark for calculating the residual values of y.
1 mark for drawing the residual plot.
1 mark for interpreting the residual plot in terms of linearity.

QUESTION 3 (6 marks)

explanatory variable = age of photocopier in years

response variable = value in dollars

$$b = r\frac{s_y}{s_x}$$

$$\begin{aligned} r &= b\frac{s_x}{s_y} \\ &= -10 \times \frac{1.5811}{16.2727} \\ &= -0.9716 \end{aligned}$$

$$\begin{aligned} \text{coefficient of determination} &= r^2 \\ &= (-0.9716)^2 \\ &= 0.9440 \\ &= 0.9440 \times 100 \text{ (convert to percentage)} \\ &= 94.40\% \end{aligned}$$

Therefore, 94.40% of the variation in the value of the photocopier can be explained by the variation in its age. 5.6% of the variation in the value of the photocopier can be explained by other factors.

[6 marks]

1 mark for determining the explanatory and response variables.

Note: This may be implied by subsequent working.

1 mark for rearranging the gradient and correlation coefficient formula for r.

1 mark for substituting into the rearranged formula.

1 mark for calculating the value of r.

1 mark for finding the coefficient of determination.

1 mark for interpreting the coefficient of determination in terms of the variables.

QUESTION 4 (7 marks)

$\theta = 21°34'$

$D = 18\,743$

$$D = 111.2\cos\theta \times \text{angular distance}$$

$$18\,743 = 111.2\cos(21°34') \times \text{angular distance}$$

$$\text{angular distance} = \frac{18\,743}{111.2\cos(21°34')}$$

$$= 181°14'$$

$$181°14' + 68°23' = 249°37'$$

$$360° - 249°37' = 110°23' \text{ W}$$

Therefore, the pilot is now at 21°34′ S, 110°23′ W.

$D = 6358$

$$D = 111.2 \times \text{angular distance}$$

$$6358 = 111.2 \times \text{angular distance}$$

$$\text{angular distance} = \frac{6358}{111.2}$$

$$= 57°11'$$

$$\text{latitude} = 57°11' - 21°34'$$

$$= 35°37' \text{ N}$$

Therefore, the coordinates of the final position of the pilot are 35°37′ N, 110°23′ W.

[7 marks]
1 mark for substituting into the distance cos formula.
1 mark for calculating the angular distance.
1 mark for determining the longitude of the second position.
1 mark for substituting into the distance formula.
1 mark for calculating the angular distance.
1 mark for calculating the latitude.
1 mark for stating the coordinates of the final position.

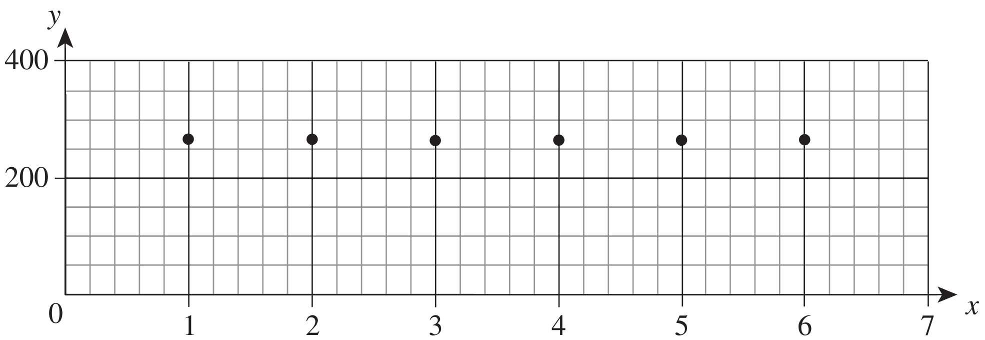
QUESTION 5 (6 marks)

$$6 \text{ monthly average} = \frac{157+184+126+382+400+335}{6}$$
$$= 264$$

Month	1	2	3
Raw sales	257	384	126
Seasonal indices	$\frac{257}{264} = 0.97$	$\frac{384}{264} = 1.45$	$\frac{126}{264} = 0.48$
Deseasonalised data	$\frac{257}{0.97} = 264.95$	$\frac{384}{1.45} = 264.83$	$\frac{126}{0.48} = 262.5$

Month	4	5	6
Raw sales	282	400	135
Seasonal indices	$\frac{282}{264} = 1.07$	$\frac{400}{264} = 1.52$	$\frac{135}{264} = 0.51$
Deseasonalised data	$\frac{282}{1.07} = 263.55$	$\frac{400}{1.52} = 263.16$	$\frac{135}{0.51} = 264.71$

$y = 264.47 - 0.15x$



The time series plot shows that, when it is deseasonalised, the data is very steady with no obvious positive or negative gradient.

[6 marks]
1 mark for determining the 6 monthly average.
1 mark for determining the seasonal indices.
1 mark for determining the deseasonalised data.
1 mark for determining the least-squares equation for the data.
1 mark for drawing the time series plot using the deseasonalised data.
1 mark for describing the trend.
Note: Accept alternative solutions based on rounding.

QUESTION 6 (7 marks)

$$A = \frac{M}{i}$$

$$\begin{aligned} M &= 150\,000 \times \frac{\frac{3.86}{12}}{100} \\ &= \$482.50 \end{aligned}$$

$$\begin{aligned} i &= \frac{\frac{2.57}{12}}{100} \\ &= 0.00214166\ldots \end{aligned}$$

$$\begin{aligned} n &= 5 \times 12 \\ &= 60 \text{ months} \end{aligned}$$

$$\begin{aligned} A_{FV} &= M\left(\frac{(1+i)^n - 1}{i}\right) \\ &= 482.50\left(\frac{\left(1+\frac{\frac{2.57}{12}}{100}\right)^{60} - 1}{\frac{\frac{2.57}{12}}{100}}\right) \\ &= \$30\,857.14 \end{aligned}$$

Therefore, the future value of the annuity is \$30 857.14.

Evaluation:

The future value can also be determined using a recurrence relation.

$$A_1 = 482.50,\ A_{n+1} = \left(1+\frac{\frac{2.57}{12}}{100}\right)A_n + 482.50$$

$$A_2 = 966.03$$

$$\vdots$$

$$A_{60} = 30\,857.14$$

[7 marks]

1 mark for using the perpetuity formula.
1 mark for calculating the monthly payment from the perpetuity.
1 mark for calculating the values of i and n for the annuity.
1 mark for substituting into the correct annuity formula.
1 mark for calculating the future value of the annuity.
1 mark for using a recurrence rlation to evaluate the reasonableness of the solution.
1 mark for calculating the value of the recurrence relation.

QUESTION 7 (5 marks)

Writing the table in matrix form gives:

$$\begin{bmatrix} 18 & 78 & 97 \\ 22 & 12 & 48 \\ 20 & 64 & 99 \end{bmatrix}$$

Identifying the lowest value in each row (row 1: 18, row 2: 12, row 3: 20) and subtracting from each row gives:

$$\begin{bmatrix} 0 & 60 & 79 \\ 10 & 0 & 36 \\ 0 & 44 & 79 \end{bmatrix}$$

Only two edges are needed to cover all the zeros, so commencing column reduction using the lowest value in each column (column 1: 0, column 2: 0, column 3: 36) gives:

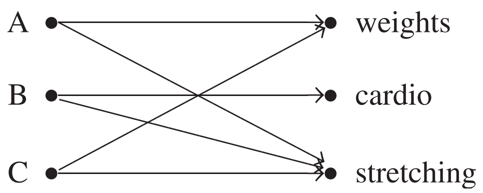
$$\begin{bmatrix} 0 & 60 & 43 \\ 10 & 0 & 0 \\ 0 & 44 & 43 \end{bmatrix}$$

Still, only two lines needed to cover all zeros. Therefore, creating additional zeros by subtracting the lowest value (43) from remaining values and adding 43 to the cross-over of the two lines gives:

$$\begin{bmatrix} 0 & 17 & 0 \\ 53 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Three lines are now required to cover all zeros, so it is ready to allocate to tasks.

Each zero corresponds to an allocation as shown in the bipartite graph.



Using the bipartite graph to allocate tasks gives:

Trainer B must perform cardio, trainer C must perform stretching and trainer A must perform weights.

$$\begin{bmatrix} 18 & 78 & 97 \\ 22 & 12 & 48 \\ 20 & 64 & 99 \end{bmatrix}$$

$$\begin{aligned} \text{minimum cost} &= 18 + 12 + 99 \\ &= \$129 \end{aligned}$$

Therefore, the minimum cost for the project will be \$129 if trainer A performs weights, trainer B performs cardio and trainer C performs stretching.

[5 marks]
1 mark for reducing each row.
1 mark for reducing each column.
1 mark for creating additional zeros.
1 mark for allocating each task to one person.
1 mark for determining the minimum cost.