Neap

Trial Examination 2021

Suggested solutions

QCE General Mathematics Units 3&4

Paper 2

Copyright © 2021 Neap Education Pty Ltd ABN 43 634 499 791 91a Orrong Cres Caulfield North VIC 3161 Tel: (03) 9639 4318 TEQGM34_SS_P2_2021

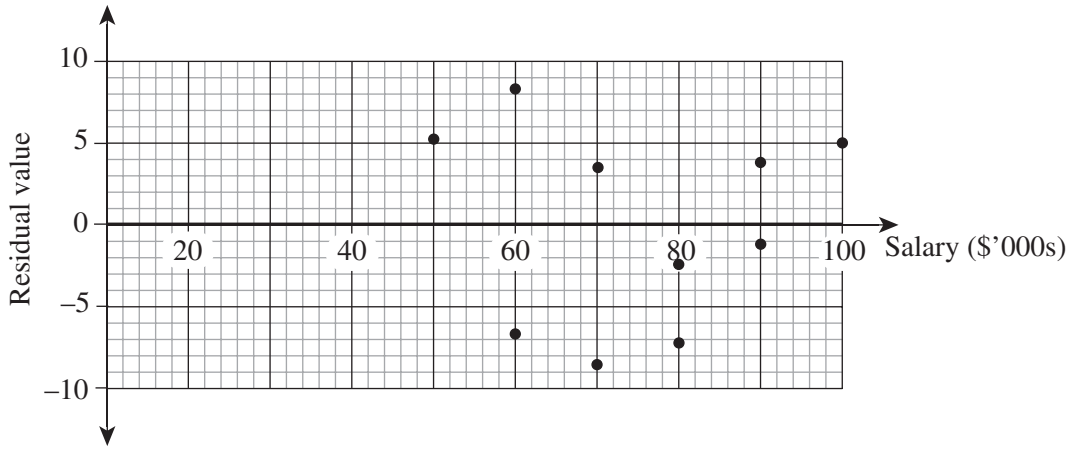
SECTION 1

QUESTION 1 (4 marks)

Salary ($1000's)	Amount paid for car ($1000's)	Predicted value	Residual value
50	6	0.97	5.03
60	3	9.75	–6.75
70	10	18.53	–8.53
80	20	27.31	–7.31
90	40	36.09	3.91
100	50	44.87	5.13
60	18	9.75	8.25
70	22	18.53	3.47
80	25	27.31	–2.31
90	35	36.09	–1.09

For example:

$$\text{predicted value of } y = -42.933 + 0.878 \times 50$$
$$= 0.97$$
$$\text{residual value} = \text{actual value of } y - \text{predicted value of } y$$
$$= 6 - 0.97$$
$$= 5.03$$



The residual plot is randomly scattered across the x-axis which suggests the presence of a linear association between a person's salary and how much they paid for their car.

[4 marks]
1 mark for calculating the predicted values of y.
1 mark for calculating the residual values.
1 mark for plotting the residual values on the graph.
1 mark for correctly interpreting the residual plot in terms of linearity.

QUESTION 2 (7 marks)

$$\text{2019 yearly average} = \frac{38\,540 + 32\,470 + 50\,490 + 45\,270}{4}$$
$$= \frac{166\,770}{4}$$
$$= 41\,692.5$$

2019 quarter	1	2	3	4
Seasonal indices	$\frac{38\,540}{41\,692.5} = 0.924$	$\frac{32\,470}{41\,692.5} = 0.779$	$\frac{50\,490}{41\,692.5} = 1.211$	$\frac{45\,270}{41\,692.5} = 1.086$

$$\text{2020 yearly average} = \frac{41\,320 + 33\,640 + 52\,340 + 46\,690}{4}$$
$$= \frac{173\,990}{4}$$
$$= 43\,497.5$$

2020 quarter	1	2	3	4
Seasonal indices	$\frac{41\,320}{43\,497.5} = 0.950$	$\frac{33\,640}{43\,497.5} = 0.773$	$\frac{52\,340}{43\,497.5} = 1.203$	$\frac{46\,690}{43\,497.5} = 1.073$

$$\text{2021 yearly average} = \frac{43\,800 + 35\,680 + 55\,270 + 47\,260}{4}$$
$$= \frac{182\,010}{4}$$
$$= 45\,502.5$$

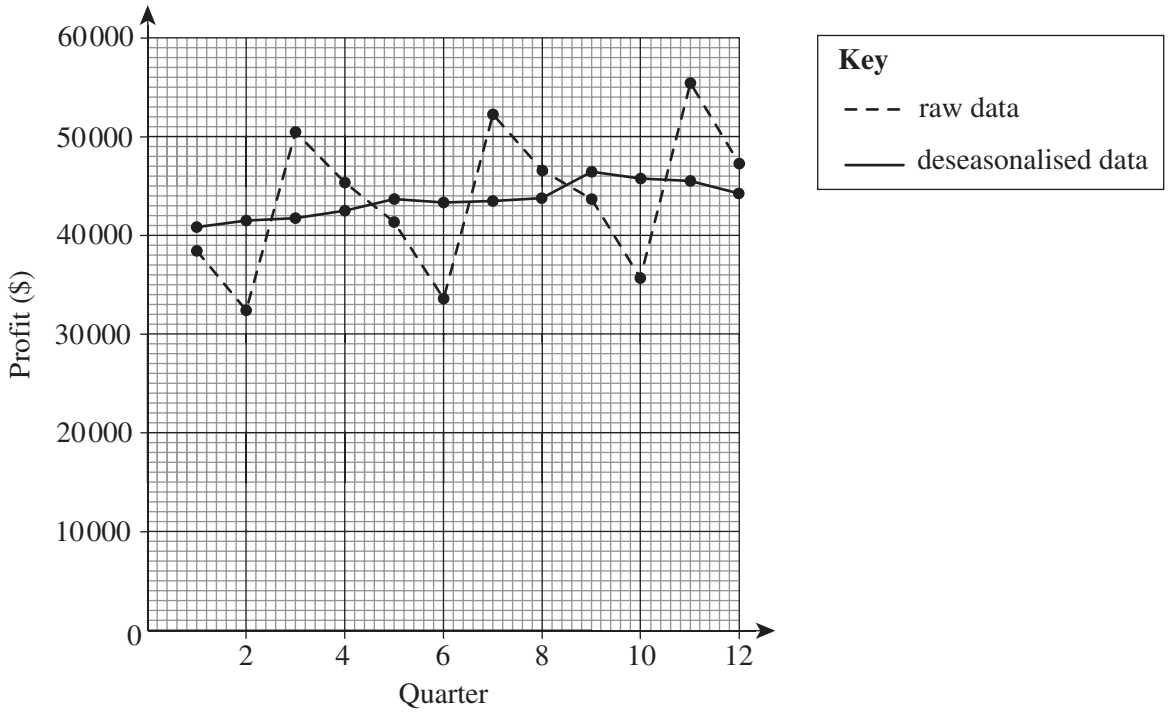
2021 quarter	1	2	3	4
Seasonal indices	$\frac{43\,800}{45\,502.5} = 0.963$	$\frac{35\,680}{45\,502.5} = 0.784$	$\frac{55\,270}{45\,502.5} = 1.215$	$\frac{47\,260}{45\,502.5} = 1.039$

2019, 2020 and 2021 average seasonal indices:

Quarter	1	2	3	4
Average seasonal indices	$\frac{0.924 + 0.950 + 0.963}{3} = 0.946$	$\frac{0.779 + 0.773 + 0.784}{3} = 0.779$	$\frac{1.211 + 1.203 + 1.215}{3} = 1.210$	$\frac{1.086 + 1.073 + 1.039}{3} = 1.066$

2019, 2020 and 2021 deseasonalised values:

Quarter	1	2	3	4
2019 deseasonalised values	$\frac{38\,540}{0.946} = 40\,740$	$\frac{32\,470}{0.779} = 41\,682$	$\frac{50\,490}{1.210} = 41\,727$	$\frac{45\,270}{1.066} = 42\,467$
2020 deseasonalised values	$\frac{41\,320}{0.946} = 43\,679$	$\frac{33\,640}{0.779} = 43\,184$	$\frac{52\,340}{1.210} = 43\,256$	$\frac{46\,690}{1.066} = 43\,799$
2021 deseasonalised values	$\frac{43\,800}{0.946} = 46\,300$	$\frac{35\,680}{0.779} = 45\,802$	$\frac{55\,270}{1.210} = 45\,678$	$\frac{47\,260}{1.066} = 44\,334$



The time series graph shows a long-term increasing trend.

[7 marks]
1 mark for correctly determining the yearly averages for 2019, 2020 and 2021.
1 mark for determining the seasonal indices for each quarter over the 3 years.
1 mark for determining the average seasonal indices over the 3 years.
1 mark for determining the deseasonalised values for each quarter over the 3 years.
1 mark for accurately plotting the deseasonalised data on the graph.
1 mark for accurately plotting the raw data on the graph.
1 mark for identifying a long-term increasing trend.
Note: Final answer given to three decimal places or more is acceptable.

QUESTION 3 (5 marks)

$t_n = 250\,000$

$t_1 = 5$

$r = 115\%$

$= \frac{115}{100}$

$= 1.15$

$t_n = t_1 r^{(n-1)}$

$250\,000 = 5 \times 1.15^{(n-1)}$

$\frac{250\,000}{5} = 1.15^{(n-1)}$

$50\,000 = 1.15^{(n-1)}$

Find n by using trial and error:

$n = 20$	$1.15^{(20-1)} = 14.23$	too low
$n = 100$	$1.15^{(100-1)} = 1\,021\,142.13$	too high
$n = 50$	$1.15^{(50-1)} = 942.31$	too low
$n = 90$	$1.15^{(90-1)} = 252\,410.72$	too high
$n = 70$	$1.15^{(70-1)} = 152\,422.37$	too low
$n = 80$	$1.15^{(80-1)} = 62\,392.07$	too high
$n = 78$	$1.15^{(78-1)} = 47\,177.37$	too low
$n = 79$	$1.15^{(79-1)} = 54\,253.97$	just over

Alternative solution (using recursive rule):

$t_0 = 5,\ t_{n+1} = 1.15 \times t_n$

$t_0 = 5$

$t_1 = 1.15 \times 5 = 5.75$

$t_2 = 1.15 \times 5.75 = 6.6125$

$t_3 = 1.15 \times 6.6125 = 7.604375$

$\vdots$

$t_{77} = 1.15 \times 205\,118.99 = 235\,886.83$

$t_{78} = 1.15 \times 235\,886.83 = 271\,269.86$

$t_{79} = 1.15 \times 271\,269.86 = 311\,960.34$

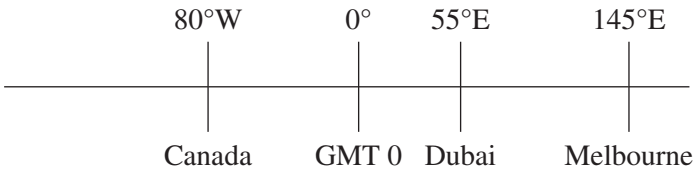
Therefore, it will take 78 hours to grow over 250 000 bacteria on the dish.

[5 marks]
1 mark for correctly identifying the parameters t_1, t_n and r.
Note: This mark may be implied by subsequent working.
1 mark for correctly substituting values into the appropriate model.
2 marks for determining the n-value.
1 mark for stating a reasonable answer rounded to the nearest hour.

QUESTION 4 (6 marks)

The local time in Melbourne at the start of the flight is 3:30 pm on Friday.

1 hour = 15° longitude



Time difference between Melbourne and Dubai:

$145° - 55° = 90°$

$\frac{90°}{15°} = 6$ hours

Thus, Melbourne is 6 hours ahead of Dubai.

Local time in Dubai at flight takeoff:

3:30 pm Friday – 6 hours = 9:30 am Friday

9:30 am Friday + 14 hours = 11:30 pm Friday local time Dubai

Time difference between Canada and Dubai:

$80° + 55° = 135°$

$\frac{135°}{55°} = 9$ hours

Thus, Canada is 9 hours behind Dubai.

Local time in Canada when Adam lands:

11:30 pm Friday – 9 hours = 2:30 pm Friday

As Adam will be able to make the phone call at 2:30 pm Canada local time, he can make the call before the office closes.

[6 marks]

1 mark for correctly calculating the time difference between Melbourne and Dubai.
1 mark for correctly subtracting the time difference.
1 mark for correctly adding the travel time.
1 mark for correctly calculating the time difference between Canada and Dubai.
1 mark for correctly subtracting the time difference.
1 mark for determining that the phone call can be made before the office closes.

QUESTION 5 (8 marks)

First year:

$M = 285$

$i = 2.35\%$

$= \frac{\frac{2.35}{52}}{100}$

$= 0.00045192307\ldots$

$n = 1 \text{ year} \times 52$

$= 52$

$A_{FV} = M\left(\frac{(1+i)^n - 1}{i}\right)$

$= 285\left(\frac{(1+0.00045192309\ldots)^{52} - 1}{0.00045192309\ldots}\right)$

$= 285 \times 52.60378868\ldots$

$= 14\,992.07977\ldots$

Second year to fifth year:

$M = 1386$

$i = 2.21\%$

$= \frac{\frac{2.21}{12}}{100}$

$= 0.00184166666\ldots$

$n = 4 \text{ years} \times 12$

$= 48$

$A_{FV} = M\left(\frac{(1+i)^n - 1}{i}\right)$

$= 1386\left(\frac{(1+0.00184166666\ldots)^{48} - 1}{0.00184166666\ldots}\right)$

$= 1386 \times 50.13729886\ldots$

$= 69\,490.29622\ldots$

Compound interest on first year amount from second year to fifth year:

$i = 2.21\%$

$= \frac{\frac{2.21}{12}}{100}$

$= 0.00184166666\ldots$

$n = 4 \text{ years} \times 12$

$= 48$

$A = P(1+i)^n$

$= 14992.07977\ldots(1+0.00184166666\ldots)^{48}$

$= 16376.39133\ldots$

Total amount in annuity account:

$16\,376.39133\ldots + 69\,490.29622\ldots = 85\,866.68755\ldots$

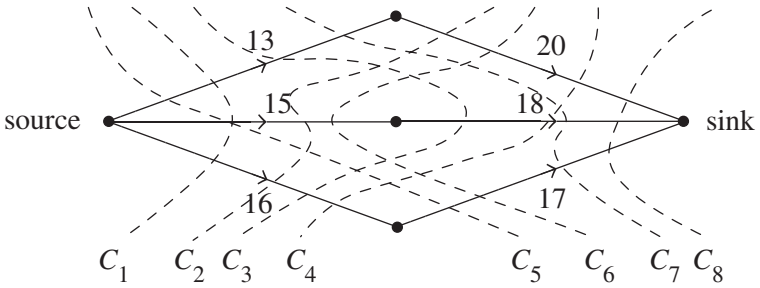
Anna will have saved \$85 866.69 in her annuity account after 5 years. As this is more than the \$85 000 she requires, she will be able to afford her holiday.

[8 marks]

1 mark for correctly determining the i and n values for the first year.
Note: This mark may be implied by subsequent working.
1 mark for correctly selecting the appropriate future value annuity rule.
1 mark for determining the future value of the annuity after one year.
Note: This mark may be implied by subsequent working.
1 mark for determining the compound interest earned on the first year of payments for the next four years.

1 mark for correctly determining the i and n values for the second to fifth year.
Note: This mark may be implied by subsequent working
1 mark for determining the future value of the annuity after four years.
Note: This mark may be implied by subsequent working.
1 mark for determining the total amount in the account.
1 mark for determining that Anna can afford her holiday and provides reasoning.

QUESTION 6 (5 marks)



$C_1 = 13 + 15 + 16$
$= 44$
$C_2 = 20 + 15 + 16$
$= 51$
$C_3 = 13 + 18 + 16$
$= 47$
$C_4 = 20 + 18 + 16$
$= 54$
$C_5 = 13 + 15 + 17$
$= 45$
$C_6 = 20 + 15 + 17$
$= 52$
$C_7 = 13 + 18 + 17$
$= 48$
$C_8 = 20 + 18 + 17$
$= 55$

Minimum cut, maximum flow = 44

cost of minimum cut + labour hire $= (44 \times 215) + 5380$
$= \$14\,840$

Therefore, the electrician can complete the installation within the quoted amount of $15 000 with $160 to spare.

[5 marks]
2 marks for identifying all of the cuts in the diagram.
1 mark for identifying the value of the minimum cut, maximum flow.
1 mark for calculating the cost of the minimum cut plus labour hire.
1 mark for determining that the electrician can complete the installation within the quoted amount.

QUESTION 7 (5 marks)

Matrix form:

$$\begin{matrix} 61 & 81 & 19 \\ 65 & 78 & 50 \\ 66 & 82 & 34 \end{matrix}$$

Identify the lowest value in each row and subtract from each row:

$R_1 - 19, R_2 - 50, R_3 - 34$

$$\begin{matrix} 42 & 62 & \mathbf{0} \\ 15 & 28 & \mathbf{0} \\ 32 & 48 & \mathbf{0} \end{matrix}$$

Only one line is needed to cover all the zeros, so commence column reduction:

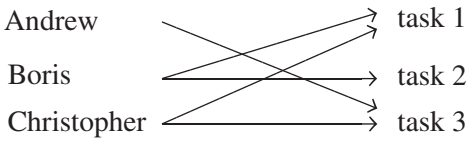
$C_1 - 15, C_2 - 28$

$$\begin{matrix} 27 & 34 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 17 & 20 & \mathbf{0} \end{matrix}$$

Only two lines are needed to cover all zeros. Therefore, create additional zeros by subtracting the lowest value (17) from remaining values and add 17 to the cross-over of the two lines.

$$\begin{matrix} 10 & 17 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 17 \\ \mathbf{0} & 3 & \mathbf{0} \end{matrix}$$

Three lines are now required to cover all zeros, so tasks can be allocated. Each zero corresponds to an allocation.



Use the bipartite graph to allocate tasks:

Boris should perform task 2, Christopher should perform task 1 and Andrew should perform task 3.

$$\begin{matrix} 61 & 81 & \mathbf{19} \\ 65 & \mathbf{78} & 50 \\ \mathbf{66} & 82 & 34 \end{matrix}$$

$$\begin{aligned} \text{minimum hours} &= 66 + 78 + 19 \\ &= 163 \text{ hours} \end{aligned}$$

The minimum hours for the project will be 163 if Andrew performs task 3, Boris performs task 2 and Christopher performs task 1.

[5 marks]
1 mark for correctly reducing each row.
1 mark for correctly reducing each column.
1 mark for correctly creating additional zeros.
1 mark for allocating each task to one person.
1 mark for determining the minimum hours.

